Towards Reconfigurable Phase Changing Articulated Drives for Robot Walking

Jasmine Yang, Reid Kaluger, Mina Hafez and Nina Robson

Abstract Contemporary robot leg mechanisms are designed with upper and lower segments actuated by motor drives that provide locomotion. Oscillation of these motors back and forth is required to guide the foot trajectory, which, in coordination with the other legs, performs the desired walking motion. The continual acceleration and deceleration of these motors waste energy, and experiments show this loss can be reduced through using articulated drives in which the motors rotate in one direction. With the goal to inspire the development of novel robust robotic systems characterized by small number of actuators and a variety of motions, this paper builds up on our recent efforts in utilizing phase changing drives for realizing a spectrum of motions. Here we present the analysis of a two actuators phase changing drive model, utilized in the creation of a multi-bar linkage leg for a walking robotic platform. Preliminary results on controlling the position of the actuators for a desired performance is presented.

1 Introduction

The Theo Jansen eight-bar leg mechanism is an important example of an articulated system for walking applications. Theo Jansen's Strandbeest [1] has a single actuator rotating in one direction and articulated drives for 20 pairs of legs that guide the foot trajectories. Similar to Strandbeest, the RHex robot [2, 3] has a single motor drive for each of six semi-circular feet. These motors spin the feet in one direction to drive the robot forward over a variety of terrain. A more usual approach is the Boston Dynamics Spot robot [4], which has four legs with three actuators for each leg, all of which must accelerate and decelerate in coordination for each gait cycle to provide locomotion. Nansai et al [5] present a reconfigurable design, where a Theo Jansen leg has variable hardware morphology obtained by parametric change of its links. Komoda and Wagatsuma [6] propose an extended version of the Theo Jansen linkage for climbing over bumps.

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Articulated drives are known to provide mechanical advantage that increases the performance of leg mechanisms [7, 8, 9]. Unfortunately, existing designs for articulated drives yield single purpose devices that are not appropriate for robotic applications [10]. As robotic systems become increasingly valuable in real world environments, new methods for developing energy efficient articulated drives is needed to transition to systems that embody a spectrum of motion behaviors while maintaining efficiency and high performance. This paper studies the analysis of the reconfigurable driving mechanism used in a recently developed robotic platform [11], shown in Figure 1, able to obtain a spectrum of motions. This is achieved by introducing structural variations within the articulated driving mechanism which yield parameterized output movement, a critical capability that has not been utilized in current methods.

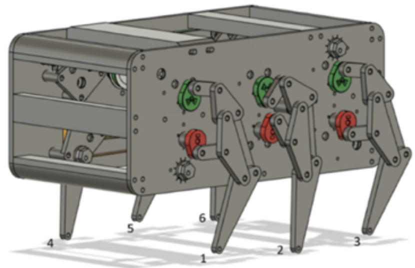


Fig. 1. Six-legged walking platform. Each leg incorporates an articulated drive actuated by two independently controlled motors.

2 The Phase-Changing Driving Mechanism

Figure 2 on the left shows a prototype of the multi-bar articulated leg of the six-legged walking platform [11, 12]. An expanded view of the dual actuator phase changing articulated leg drive is given in Figure 2 on the right. The drive can be modeled as a four-bar linkage OO'AB, with a base link OO' and a varying coupler length h, shown in Figure 3. Note, that the Grashoff's condition is met for each different coupler length to allow continuous rotation of the input actuators. The drive is actuated by two independently controlled motors, located at points O and O'. The coupler variation allows for changing the phase between the two motors, yielding different coupler curves that result in a spectrum of motions at the foot.

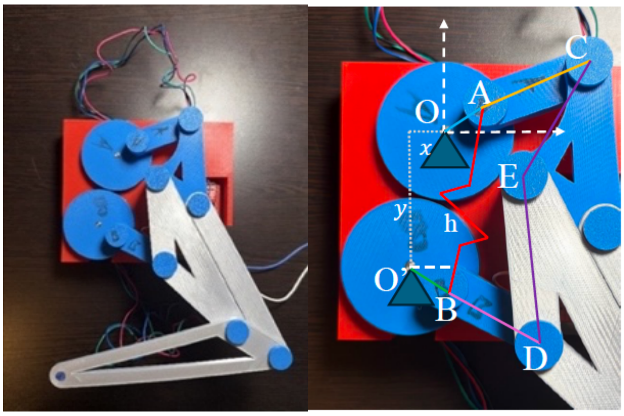


Fig. 2. The phase changing drive-leg system (left). Expanded view of the phase-changing drive (right).

The two actuators work together to achieve a specific coupler path with each providing the necessary torque to control the motion of its respective link. The articulated drive is part of two four-bar chains: OECA which drives the rocking motion of the upper leg and leaves the lower leg in the same orientation and O'EDB which drives the rocking motion of the lower leg and keeps the knee joint fixed. To produce a specific walking trajectory crank OA rotates clockwise, while O'B rotates counter-clockwise. To change the foot trajectory, the lower driving crank O'B can be offset by a specific phase angle.

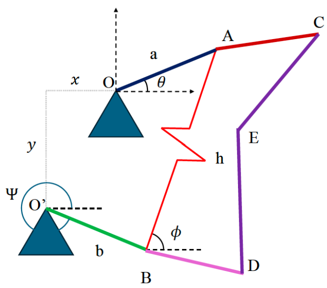


Fig. 3. Phase changing drive modeled as a four-bar linkage with varying coupler length.

In what follows, we present the kinematic analysis of the phase-changing drive.

2.1 Constraint Equation of the Phase Changing Drive Modeled as a Four-Bar Linkage with a Varying Coupler Length

Consider the cranks OA and O' B from the four-bar linkage model discussed above. Assuming that **A** and **B** can be connected to each other by a virtual link AB of varying coupler length h and O is the origin of the fixed frame of the four-bar OO'AB, we can write the constraint equation as:

$$(\boldsymbol{B} - \boldsymbol{A}) \cdot (\boldsymbol{B} - \boldsymbol{A}) = h^2 \tag{1}$$

$$\begin{Bmatrix} a \cdot \cos\theta \\ a \cdot \sin\theta \end{Bmatrix} = \boldsymbol{A} \tag{2}$$

$$\begin{Bmatrix} -x + b \cdot \cos\psi \\ -y + b \cdot \sin\psi \end{Bmatrix} = \boldsymbol{B} \tag{3}$$

By substituting equations (2) and (3) into (1), the length of the coupler is:

$$\sqrt{(-x + b\cos\psi - acos\theta)^2 + (-y + b\sin\psi - asin\theta)^2} = |h| \tag{4}$$

Table 1 shows results on the rate of change of the coupler length h for the phase changing drive model shown in Figures 2 and 3 when both θ and ψ vary at 90° increments. Note that a = 13.77 mm, b = 13.77 mm, x = 7.70 mm, y = 30.8 mm and that the two inputs can be controlled independently. Thus, the minimum and maximum coupler limits for the model shown in Figure 3 are 8.369 and 58.86 mm.

Table 1. Example of the change in length of h [mm] as θ (vertical) and ψ (horizontal) vary at increments of 90° over time.

Theta/Psi	0°	90°	180°	270°	360°
0°	31.76732	27.41648	46.81592	49.48969	31.76732
90°	45.00126	31.76732	49.48969	58.86578	45.00126
180°	36.65376	18.09827	31.76732	45.00126	36.65376
270°	18.09827	8.369492	27.41648	31.76732	18.09827
360°	31.76732	27.41648	46.81592	49.48969	31.76732

2.2 Loop Equations of the Phase Changing Drive Model

Next, the position loop vector equation for the phase changing drive, modeled as a four-bar linkage with a changing coupler length can be used to solve for the coupler angle ϕ:

$$[\boldsymbol{A}] + [\boldsymbol{B} - \boldsymbol{A}] = [\boldsymbol{C}] + [\boldsymbol{B}] \tag{5}$$

$$\begin{Bmatrix} hcos(\phi) \\ hsin(\phi) \end{Bmatrix} = \begin{Bmatrix} -x \\ -y \end{Bmatrix} + \begin{Bmatrix} b\cos\psi \\ b\sin\psi \end{Bmatrix} - \begin{Bmatrix} a\cos\theta \\ a\sin\theta \end{Bmatrix} \tag{6}$$

From here, the coupler angle ϕ can be found as:

$$\phi = \tan^{-1}\left(\frac{-y + b\sin\psi - a\sin\theta}{-x + b\cos\psi - a\cos\theta}\right) \tag{7}$$

Table 2 shows results for the changes in the coupler angle ϕ as a function of both θ and ψ for the drive shown in Figures 2 and 3. Note, that a= 13.77 mm, b=13.77 mm, x= 7.70 mm, y= 30.8 mm.

Table 2. Change in length of ϕ as θ (vertical) and ψ (horizontal) change over time.

Theta/Psi	0°	90°	180°	270°	360°
0°	75.97251	38.45419	41.17211	64.28927	75.97251
90°	-82.248	75.97251	64.28927	82.48383	-82.248
180°	-57.2291	-70.4037	75.97251	-82.248	-57.2291
270°	-70.4037	23.07274	38.45419	75.97251	-70.4037
360°	75.97251	38.45419	41.17211	64.28927	75.97251

The velocity loop equations of the modeled four-bar linkage drive can be used to simultaneously solve for the varying coupler angle and coupler length:

$$\begin{bmatrix} -\,asin\,\theta \\ acos\,\theta \end{bmatrix}\dot{\theta} + \begin{bmatrix} -h\,sin\,\phi \\ h\,cos\,\phi \end{bmatrix}\dot{\phi} + \begin{bmatrix} cos\,\phi \\ sin\,\phi \end{bmatrix}\dot{h} = \begin{bmatrix} -x \\ y \end{bmatrix} + \begin{bmatrix} -b\,sin\,\psi \\ b\,cos\,\psi \end{bmatrix}\dot{\psi} \tag{8}$$

$$\dot{\phi} = \frac{sin\phi\left(-x - bsin\psi\dot{\psi} + asin\theta\dot{\theta}\right) - cos\phi\left(-y + bcos\psi\dot{\psi} - acos\theta\dot{\theta}\right)}{-h} \tag{9}$$

$$\dot{h} = sin\phi\left(-y + bcos\psi\dot{\psi} - acos\theta\dot{\theta}\right) + cos\phi\left(-x - bsin\psi\dot{\psi} + asin\theta\dot{\theta}\right) \tag{10}$$

In a similar fashion, the acceleration loop equations can be derived to simultaneously solve for the angular and linear acceleration.

2.3 Mechanical Advantage of the Phase-Changing Drive Modeled as a Four Bar Linkage with Varying Coupler Length

Unlike a standard four-bar linkage, a single Mechanical Advantage (MA) value for the articulated drive is meaningless, due to the varying coupler length and the use of two actuators. The MA becomes a system property dependent on the torque ratio between the two actuators. The combined MA is a function of the systems geometry and the relative torques applied by the two actuators. Below we present the instantaneous MA for each actuator by holding the other one still, which describe how each actuator contributes independently to the input.

The instantaneous MA for each actuator is defined as:

$$|MA_1| = \frac{a * \dot{\theta}}{\dot{h}} \tag{11}$$

$$|MA_2| = \frac{b * \dot{\psi}}{\dot{h}} \tag{12}$$

By taking the time derivative of the constraint equation, the velocity ratios $\frac{\dot{\theta}}{\dot{h}}$ and $\frac{\dot{\psi}}{\dot{h}}$ can be found resulting in the following instantaneous MA for each actuator:

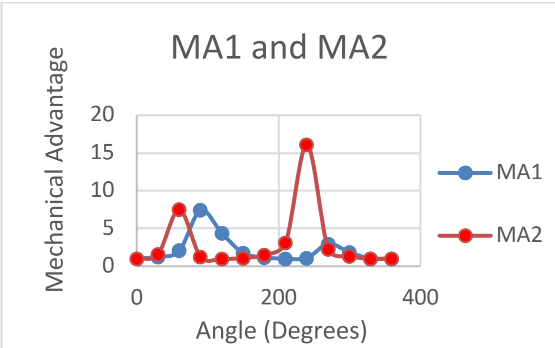
$$|MA_1| = \frac{h}{-x\sin(\theta) + b\sin(\theta) + y\cos(\theta)} \tag{13}$$

$$|MA_2| = \frac{h}{x\sin(\psi) + a\sin(\psi) - y\cos(\psi)} \tag{14}$$

From equations (13) and (14) it can be seen that MA_1 is a function of crank OA=a, while MA_2 is a function of crank O'B=b. The combined MA for the system is a relationship that depends on the coordination of the two actuators. Thus, the weighted average of the two mechanical advantages based on the relative torque input can be expressed as the combined MA:

$$|MA_{combined}| = \frac{T_{1in} * |MA_1| + T_{2in} * |MA_2|}{T_{1in} + T_{2in}} \tag{15}$$

An example result for the instantaneous MAs for each actuator for the phase changing drive model shown in Figure 3, are depicted in Figure 4. The MAs are evaluated as a function of the input angles θ and ψ at 30° increments over a full rotation. The first mechanical advantage MA_1 is plotted as a function of the input angle θ, while ψ is kept at 0 degrees. The second MA_2 is a function of the input angle ψ, while θ is kept at 0 degrees. Specifically, Figure 4 on the left shows how each actuator of the phase changing drive model contributes independently. The combined MA is shown in Figure 4 on the right for $T_{1in} = 16\ Ncm\ and\ T_{2in} = 16\ Ncm$. It can be noticed that the drive model reaches its maximum MA_1 when the input link and the coupler become colinear (θ = 101 or 281 deg) and its maximum MA_2 when the coupler link and the output link become colinear (ψ = 55 or 235 deg). Note that the drive model does not reach a toggle position, ensuring a consistent continuous path of force transmission within the system. By actively controlling the position or torque of both actuators, one can dynamically alter the Mechanical Advantage, leading to different range of possible motions and a more complex and dynamic performance.



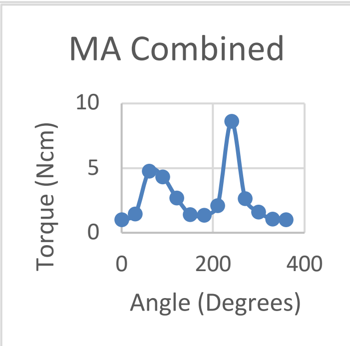


Fig. 4. Mechanical advantage MA_1 and MA_2 plotted as a function of the input angles θ and ψ, evaluated at 30° increments over a full rotation (left). Combined MA as the weighted average of MA_1 and MA_2.

Figure 5 shows an example of the foot trajectory of the leg prototype, generated using Adobe After Effects, when only one of the actuators is working with an initial offset of 270 degrees. When only the upper crank rotates, it controls the hip motion which rotates the upper leg and leaves the lower leg in the same orientation (see Figure 5 on the left). When only the lower crank rotates, it controls the knee which rotates the lower leg and keeps the knee joint fixed (see Figure 5 on the right).

Fig. 5. Foot trajectories when only the upper (left) or the lower (right) crank rotates.

Figure 6 shows an example of the trajectories of points C and D (shown in Figures 2 and 3) of the drive with an offset of 270 degrees, when only one of the actuators is working. Both trajectories follow a tilt motion as each point is located on the coupler of a separate four-bar linkage. It is worth mentioning that the trajectories geometry remains the same regardless of the offset.

Fig. 6. Trajectories of Points C (left) and D (right) with a 270 degree offset. It can be noted that regardless of the offset, the trajectories geometry remains the same.

Figure 7 presents an example of how the phase changing drive can be utilized to effectively design the kinematics of the system in real time. A variety of gaits can be produced based on introducing a phase offset. To change the phase, the motor corresponding to the lower drive crank O'B rotates to produce the appropriate offset angle while the upper drive crank is held in place. Once the appropriate phase offset has been introduced both motors rotate, resulting in a new foot trajectory.

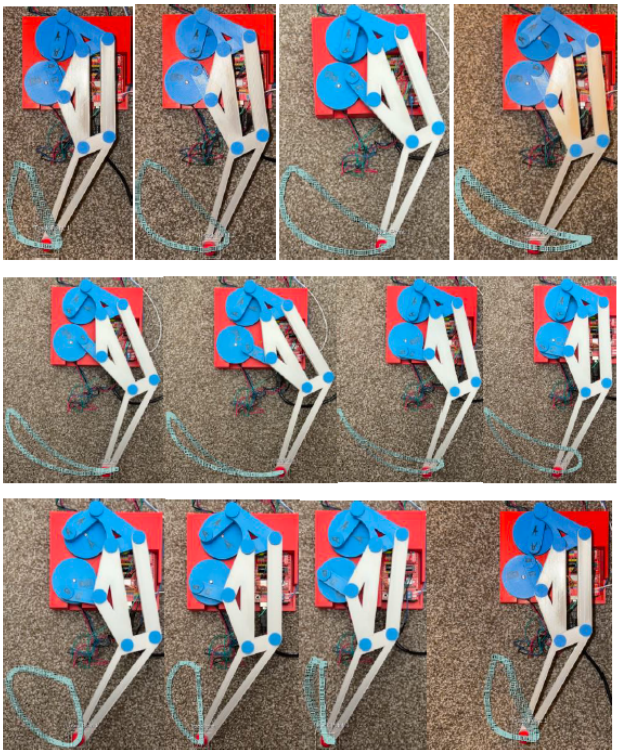


Fig. 7. Examples of various foot trajectories achieved by different phase offsets at 0°, 30°, 45°, 90° (first row), 120°, 150°, 180°, 245° (second row), 270°, 300°, 330°, 360° (third row).

3 Summary

This paper builds up on our recent efforts in utilizing phase changing drives for realizing a spectrum of motions for robotic applications. An analysis of the four-bar linkage phase changing drive consisting of two actuators is performed, ensuring that the mechanism utilizes continuity in motion. The results for the Mechanical Advantage show the contribution of each actuator as a continuous function that does not lead to the motors working against each other. Preliminary results on actively controlling the position of the actuators, leading to complex and dynamic system performance is also presented.

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