Geometry and Direct Kinematics of 3 – $\underline{C}$CC Type Parallel Manipulator

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Abstract. In this paper the methods of geometry description and direct kinematics of six – DOF three – limbed parallel manipulator (PM) of a 3-CCC type are presented, where C denotes a cylindrical joint. This PM is formed by connection of a mobile platform with a base by three dyads with cylindrical joints. Constant and variable parameters characterizing geometry of links and relative motions of elements of joints are defined. An algorithm for the PM direct kinematics by iterative method is proposed.

1 Introduction

Most of six – DOF PMs consist of six limbs [1-5]. These PMs possess the advantages of high stiffness, low inertia, and large payload capacity. However, such six – limbed fully PMs have a limited workspace and complex kinematic singularities, which are their major drawbacks. Therefore in robotics literature a great interest to six - DOF PMs with three limbs (tripods) is observed. The following types of six – DOF three - limbed PMs are known: 3-URS [6], 3-ESR [7], 3-RRPS [8], 3-RES [9-11], 3-PPSP [12], 3-CRS [13], 3-UPU, 3-URU[14, 15], and 3-$\underline{C}$CC [16-19].

In the kinematic models (architectures) of 3-$\underline{C}$CC type PMs discussed in [16–18], and the double-triangular parallel manipulators in [19], the middle cylindrical kinematic pair of each of the three CCC-type legs represents an extended analogue of a prismatic pair. In this configuration, the link undergoing translational motion also performs a rotational motion about its own axis. Furthermore, the direction of the link connecting to the cylindrical kinematic pair located on the mobile platform coincides with the direction of the rotation axis of the middle cylindrical kinematic pair. In our work, we investigate a 3-$\underline{C}$CC type PM with an arbitrary angle between

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these two directions. A Republic of Kazakhstan patent [20] has been obtained for this novel 3-C̲CC type PM.

This PM is formed by connection of a mobile platform 3 with a base 0 by three spatial dyads *ABC*, *DEF* and *GHI* of type CCC (Fig.1). Each of spatial dyads of type CCC do not impose constraints on motion of the mobile platform, and six – DOF of the mobile platform are remained. In the considered PM the joints *A*, *F* and *I* are active joints, and the joints *B*, *C*, *D*, *E*, *G* and *H* are passive joints. Six variable parameters s_7, θ_7, s_8, θ_8, s_9, θ_9 of active joints *A*, *F* and *I* are the generalized coordinates. The results of singularity analysis of the PM 3-C̲CC are presented in [21]. In this paper a geometry of this PM is described and its direct kinematics is solved.

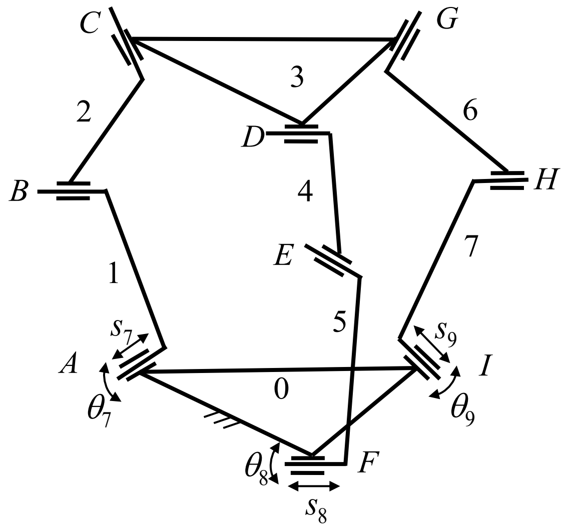

Figure 1. PM with cylindrical joints.

2 Geometry of the 3-C̲CC type PM

To describe the geometry of the PMs in work [21] two right – hand Cartesian coordinate systems $U_jV_jW_j$ and $X_kY_kZ_k$ are attached to each element of each joint. The W_j and Z_k axes of the coordinate systems $U_jV_jW_j$ and $X_kY_kZ_k$ are directed along the axes of rotation and translation motions of joints. The axes U_j and X_k are directed along a common perpendicular between the axes W_j and Z_k. In this work, a transformation matrix $\mathbf{T}_{jk}$ between the selected coordinate systems has been formulated, containing six parameters. This enables the construction of matrices for binary links and kinematic pairs, which incorporate constant and variable parameters characterizing the link geometry and the relative motions of the kinematic pair elements, respectively.

Figures 2 and 3 illustrate a CC-type binary link *jk* and a cylindrical kinematic pair *j*, respectively, along with their assigned coordinate systems.

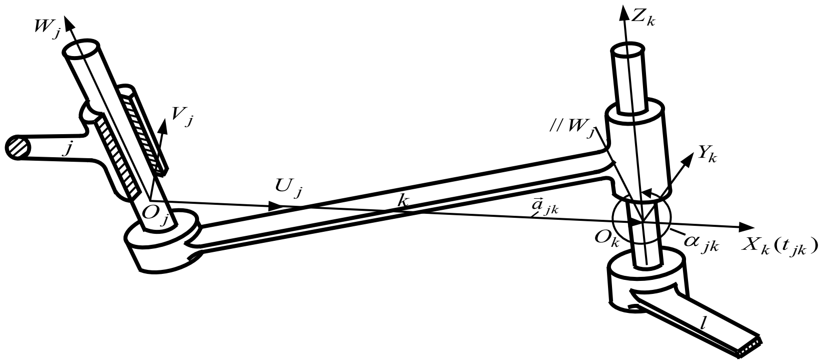

Figure 2. Binary link *jk* of type CC.

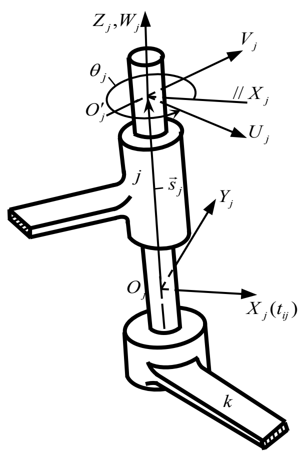

Figure 3. Cylindrical joint *j*.

At such choice of the coordinate systems $U_jV_jW_j$ and $X_kY_kZ_k$ nonzero parameters of the matrix $\mathbf{T}_{jk}$ are a_{jk} and α_{jk}. Then from the matrix $\mathbf{T}_{jk}$ we obtain a matrix of the binary link *jk* of type CC

$$\mathbf{G}_{jk}^{\mathrm{CC}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ a_{jk} & 1 & 0 & 0 \\ 0 & 0 & \cos\alpha_{jk} & -\sin\alpha_{jk} \\ 0 & 0 & \sin\alpha_{jk} & \cos\alpha_{jk} \end{bmatrix}, \tag{1}$$

where parameters a_{jk} and α_{jk} are constant, and they characterize the geometry of the binary link *jk* of type CC.

Nonzero parameters of the cylindrical joint *j* are θ_j and s_j. Then from the matrix $\mathbf{T}_{jk}$ we obtain a matrix of the cylindrical joint *j*

$$\mathbf{P}_j^{\mathrm{C}}\left(\theta_j, s_j\right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_{jk} & -\sin\theta_j & 0 \\ 0 & \sin\theta_j & \cos\theta_j & 0 \\ s_j & 0 & 0 & 1 \end{bmatrix}, \tag{2}$$

where parameters s_j and θ_j are variable, and they characterize relative translation and rotation motions of the *j* – th cylindrical joint elements.

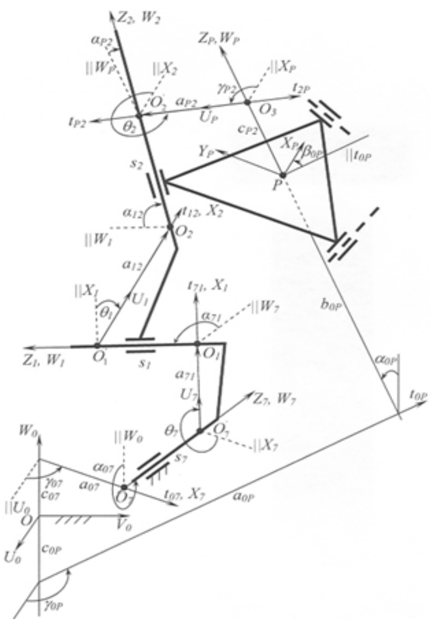

Figure 4. The first limb.

Choosing the coordinate systems $U_jV_jW_j$ and $X_kY_kZ_k$, as shown in Figs. 2 and 3, the constant and variable parameters of the PM 3-CCC have been obtained. Constant and variable parameters of the first limb *ABC* of the PM 3-CCC are shown in Fig. 4, where θ_7 and s_7 are the generalized coordinates of the active joint *A*; θ_2, s_2 and θ_3, s_3 are the variable parameters of the passive joints *B* and *C*; all other parameters are the constant parameters characterizing the geometry of links. Constant and variable parameters of two other symmetrical legs are defined similarly.

3 Direct Kinematics

In direct kinematics of the PM 3-CCC position of coordinate system $X_PY_PZ_P$ attached to the mobile platform 3 are defined with respect to the base frame $U_0V_0W_0$ by known constant geometrical parameters of the links and the generalized coordinates s_i, θ_i, (i=7,8,9).

For automation of calculation of the direct kinematics we use following designations: n is number of mobile links, m is number of input links, L is number of the closed loops, n_l is number of links in the l-th loop (l=1,2, …, L), (l,a) is index of the a-th link in the l-th loop. Dependent variable parameters of the passive joints $h_{(l,a)}^{(k)}$ are numerated 1,2,…, $n-m$, independent variable parameters of the active joints $q_{(l,a)}^{(k)}$ are numerated $n-m+1$, $n-m+2$, …, n, where k is number of DOF of the joint (l,a). For the considered PM 3-CCC n=7, m=3, L=2, $n_1=n_2=6$, ω=2. PM 3-CCC has two closed loops *ABCDEFA* and *ABCGHIA*. The passive kinematic pairs *B, C, D, E, G, H* are numbered 1, 2, 3, 4, 5, 6 respectively, and the active kinematic pairs *A, F, I* are numbered 7, 8, 9, respectively.

Let made a Table 1 of conformity between the numbers of links and joints of the PM 3-CCC.

Table 1. Conformity between the numbers of links and joints.

l	1	1	1	1	1	1	2	2	2	2	2	2
a	1	2	3	4	5	6	1	2	3	4	5	6
(l,a)	7	1	2	3	4	8	7	1	2	5	6	9

Using the Table 1 of conformity between the numbers of links and joints we can write the loop – closure equations of the loops *ABCDEFA* and *ABCGHIA*

$$\left.\begin{aligned}\mathbf{M}_{71}\cdot\mathbf{M}_{12}\cdot\mathbf{M}_{23}\cdot\mathbf{M}_{34}\cdot\mathbf{M}_{48}\cdot\mathbf{M}_{87}&=\mathbf{E}\\ \mathbf{M}_{71}\cdot\mathbf{M}_{12}\cdot\mathbf{M}_{25}\cdot\mathbf{M}_{56}\cdot\mathbf{M}_{69}\cdot\mathbf{M}_{97}&=\mathbf{E}\end{aligned}\right\}\tag{3}$$

or

$$\mathbf{M}_{l,1}\cdot\mathbf{M}_{l,2}\cdot\ldots\cdot\mathbf{M}_{l,a}\cdot\ldots\cdot\mathbf{M}_{l,n_l}=\mathbf{E};\quad l=1,2,\tag{4}$$

where $\mathbf{E}$ is a unit matrix, $\mathbf{M}_{jk}=\mathbf{P}_{jk}^{\mathrm{C}}\cdot\mathbf{G}_{jk}^{\mathrm{CC}}$, $j, k=1,2,\ldots,9$.

For the direct kinematics of the PM 3-CCC the iterative method [22] is used. According to this method unknown dependent variable parameters $h_{(l,a)}^{(k)}$ are written through their initial approaches $h_{(l,a)}^{(k)*}$ and deviations $dh_{(l,a)}^{(k)}$ by expression

$$h_{(l,a)}^{(k)}=h_{(l,a)}^{(k)*}+dh_{(l,a)}^{(k)},\tag{5}$$

where $h_{(l,a)}^{(k)}\in\theta_{l,a}, s_{l,a}$.

Each matrix is expanded as follows

$$\mathbf{M}_{l,a}\approx\mathbf{M}_{l,a}(h^{*}(l,a))+\sum_{k=1}^{2}\frac{\partial\mathbf{M}_{l,a}}{dh^{(k)}(l,a)}\,dh^{(k)}(l,a).\tag{6}$$

To differentiate the matrices $\mathbf{M}_{l,a}$ in equation (6), we introduce a matrix operator $\mathbf{Q}_{(l,a)}^{(k)}$

$$\frac{\partial\mathbf{M}_{l,a}}{\partial h^{(k)}(l,a)}\mathbf{Q}_{(l,a)}^{(k)},\tag{7}$$

where, for a cylindrical kinematic pair

$$\mathbf{Q}_{(l,a)}^{(1)}=\mathbf{Q}_{(l,a)}^{(\theta)}=\begin{bmatrix}0&0&0\\0&0&-1\\0&1&0\\0&0&0\end{bmatrix},\quad\mathbf{Q}_{(l,a)}^{(2)}=\mathbf{Q}_{(l,a)}^{(3)}=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\1&0&0&0\end{bmatrix}.\tag{8}$$

If we substitute the expression (7) of matrix $\mathbf{M}_{l,a}$ into the loop-closure matrix equations (4) and exclude the terms containing products of the form $dh(l,a)\cdot dh(l,a)$, as well as take the notations.

$$\sum_{k=1}^{2}\mathbf{B}_{l,a}^{(k)}=\mathbf{M}_{l,1}\cdot\mathbf{M}_{l,2}\cdot\ldots\cdot\mathbf{M}_{l,a-1}\cdot\sum_{k=1}^{2}\mathbf{Q}_{l,a}^{(k)}\cdot\mathbf{M}_{l,a}\cdot\ldots\cdot\mathbf{M}_{l,n_l},\tag{9}$$

$$\mathbf{D}_{l,a} = \mathbf{M}^{*}_{l,1} \cdot \mathbf{M}^{*}_{l,2} \cdot \ldots \cdot \mathbf{M}^{*}_{l,a} \cdot \ldots \cdot \mathbf{M}^{*}_{l,n_l} \qquad (10)$$

then we obtain matrix equations of the form

$$\sum_{k=1}^{2} \mathbf{B}^{(k)}_{l,a} \cdot dh^{(k)}_{(l,a)} = \mathbf{W}_{l,a}, \qquad (11)$$

where

$$\mathbf{W}_{l,a} = \mathbf{E} - \mathbf{D}_{l,a} \qquad (12)$$

In matrix equations (10) $\mathbf{M}^{*}_{l,a}$ are matrices with variables $h^{*}(l,a)$.

By solving matrix equations (11) via least-squares approximation, we determine the deviations

$$dh^{(k)}(l,a) = \left[\mathbf{B}^{\mathrm{T}}_{l,a} \cdot \mathbf{B}_{l,a}\right]^{-1} \cdot \mathbf{B}^{\mathrm{T}}_{l,a} \cdot \mathbf{W}_{l,a}. \qquad (13)$$

Adding the determined deviations to their previous values, more exact values of the passive joints variable parameters have been obtained.

Position of the coordinate system $X_P Y_P Z_P$ attached to the mobile platform 3 with respect to the base coordinate system $U_o V_o W_o$ can be defined by matrix $\mathbf{S}_P$

$$\mathbf{S}_P = \mathbf{G}_{07} \cdot \mathbf{M}_{71} \cdot \mathbf{M}_{12} \cdot \mathbf{M}_{23} \cdot \mathbf{G}_{3P}, \qquad (14)$$

where $\mathbf{G}_{07}$ is a transformation matrix between the coordinate system $X_7 Y_7 Z_7$ and the absolute coordinate system $U_0 V_0 W_0$, $\mathbf{G}_{3P}$ is a transformation matrix between the coordinate systems $X_P Y_P Z_P$ and $X_3 Y_3 Z_3$ of the mobile platform 3.

4 Conclusions

A novel six – DOF three – limbed PM 3-CCC is formed by connection of the mobile platform with the base by three spatial dyads with cylindrical joints. Constant and variable parameters of the PM 3-CCC are defined on the basis of the transformation matrix of two systems of coordinates attached to each element of each joints. Constant parameters characterize geometry of links, and variable parameters characterize relative motions of the joint elements. The loop – closure matrix equations of the PM 3-CCC are made up. An algorithm for the PM direct kinematics is proposed, using an iterative method to solve the loop-closure matrix equations. In our future work, we plan to include numerical kinematics results including singular configurations.

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