Dimensional Synthesis of Actuated Mechanisms using the Deformation Energy Method

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Abstract This paper presents a novel algorithm for the Deformation Energy Method (DEM), a category of dimensional synthesis of mechanisms which exploits the deformation of kinematic diagrams in order to produce optimized mechanisms in an efficient and timely manner. When compared to previous iterations, the present algorithm implements three new features into the DEM: the avoidance of mechanism singularities, the possible alteration of the chassis, and the optimal placement of an actuator. It was tested on a panel of scenarios comprising 4-bar and 6-bar topologies, and has shown promising results in terms of performance.

1 Introduction

When designing complex mechanisms, the obtention of an adequate kinematic diagram from a pre-established mechanism topology is a recurrent issue. This task requires specialized skills from the designer, and can be time consuming, which is why several tools have been developed to assist them in their work. This is the domain of the dimensional synthesis of mechanisms.

Early developments of this discipline can be found in Burmester's theory, elaborated in the late 19$^{\text{th}}$ century, which helps finding the dimensions of a

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4-bar mechanism able to reach a series of poses [7]. Other generation methods followed, but it was not until the development of numerical computation that generic synthesis methods (not specific to a given topology) started to appear.

As a reference, we can cite among those synthesis methods: the geometrical constraints method [9], [5], a method using an evolutionary algorithm [8], a method making use of a neural network [10], as well as the one which interests us in this paper: the Deformation Energy Method (DEM) [1], [3], [4]. This approach, developed by Avilés, considers the links of the mechanism as elastic bodies, and aims at minimizing their deformation along an imposed trajectory.

The permissiveness of the model manipulated by the DEM provides an important advantage in terms of optimization speed, but at the cost of finding approximate solutions. This can prove detrimental in terms of reliability. As is, this approach also fails to prevent mechanism singularities, which hinders its use for high numbers of bars.

In this paper, which focuses on the synthesis of planar mechanisms with revolute joints, we present three notable additions to the DEM: the possibility for the optimization scheme to modify the position of the chassis, the avoidance of mechanical singularities during the movement, and the placement of a linear actuator able to pilot the mechanism as efficiently as possible. Section 2 describes the workings of this new algorithm[1], while section 3 discusses its current results.

2 Optimization Algorithm

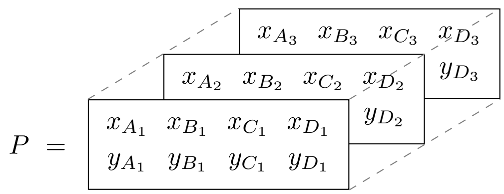
2.1 Model of the System

Since the objective of our generation is to go from a mechanism topology to a functional kinematic scheme, it is important to highlight what distinguishes both representations. The topology (or kinematic chain) of a mechanism determines the number of rigid bodies it comprises, as well as their joints. To that, its kinematic diagram adds simplified dimensions (only the distances between joints are considered), and designates among the bodies a chassis along with one or more end-effectors.

We chose to represent those various properties with a set of points $\mathbb{P}$, each corresponding to the center of a revolute joint, associated to a series of geometrical constraints. For a given configuration, a kinematic diagram containing N rigid bodies and n joints can thus be represented (in the plane) by $2 \times n$ coordinates. For a number of configurations p, we can therefore represent the mechanism with a third-order tensor of dimensions $n \times 2 \times p$,

[1] Python3 implementation available at `https://forge.inrae.fr/tscf/projects/soilfriendbot/actuated_mechanisms_dimensional_synthesis`

named $P = [p_{i,j,k}]$. A 4-bar mechanism modeled on 3 configurations would thus take the shape:



Each point $a \in \mathbb{P}$ is associated to an ordered pair of rigid bodies (i_a, j_a), of which it is the pivot point. Geometrically, two points belonging to the same rigid body must keep a fixed distance from one configuration to another. We chose to monitor those constraints with a series of indexes, referencing pairs of points. They are regrouped in the set $\mathbb{E}$, defined as follows:

$$\mathbb{E} \stackrel{\text{def}}{=} \Big\{ \{a, b\} \;\Big|\; a, b \in \mathbb{P} \;\&\; a \neq b \;\&\; (i_a = i_b \textit{ or } j_a = j_b \textit{ or } i_a = j_b \textit{ or } j_a = i_b) \Big\} \tag{1}$$

Similar indexes are used to designate a pair of points belonging to the chassis $\{a_0, b_0\} \in \mathbb{E}$, and a pair of points belonging to the effector $\{a_e, b_e\} \in \mathbb{E}$. With those four elements (P, $\mathbb{E}$, $\{a_0, b_0\}$ and $\{a_e, b_e\}$), we are able to fully describe the kinematic diagram of the mechanism.

It is also important to avoid going through singularities during the movement. Simply put, mechanism singularities (types I and II) arise when two bars linked by a revolute joint end up aligned [11]. We can thus detect them by monitoring the angles formed by those pairs of bars (figure 1). The angles are represented by trios of points, constructed from elements of $\mathbb{E}$ and regrouped in the set $\mathbb{F}$:

$$\mathbb{F} \stackrel{\text{def}}{=} \Big\{ (a, b, c) \;\Big|\; \{a, b\}, \{b, c\} \in \mathbb{E} \;\&\; a \neq c \Big\} \tag{2}$$

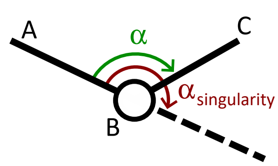


Fig. 1: Apparition of a singularity at $\alpha \equiv 0 \pmod{\pi}$

Once the geometry of the mechanism has been obtained, determining an optimal placement for its actuator can prove difficult, which is why we chose to implement it directly in our dimensional synthesis algorithm. The actuator is modeled as a bar of varying length, whose base is linked to the chassis, while its end is attached to one of the bodies of the mechanism via two new bars (the *attaches*). The optimisation of its position is done through the coordinates of the points couple $\{v, w\}$ (respectively pivot centers of the base and the end). Those coordinates are regrouped in the $2 \times 2 \times p$ tensor P_{act}, similar to P. We note $\{a^*, b^*\} \in \mathbb{E}$ the points couple to which the attaches are linked.

2.2 Global Structure

As described briefly in 1, the deformation energy method imposes a series of target poses (chosen by the user) to the effector, and tries to minimize the deformation of the mechanism through an energy criterion. Contrary to the majority of geometrical synthesis methods, its goal is thus not to get closer to a desired trajectory, but to force the trajectory and minimize the incoherence that this induces. Our algorithm is based on the approach presented in [3], which realizes this optimization by modifying the coordinates of the joints centers.

Trajectory following is not the only objective of our generation algorithm; it must also mitigate bulkiness and avoid the crossing of singularities during movement. All those criteria are expressed mathematically, and regrouped inside a loss function λ, which assigns a score to P. Minimizing $\lambda(P)$ thus allows us to find the solution corresponding best to our preferences.

The spatial restrictions of the mechanism are not integrated into λ. They are instead represented by a bounding box, which limits the research area of the optimization. a_0 and b_0 have their own bounding boxes, in order to account for additional design constraints.

Since $\lambda(P)$ presents numerous local minima, we opted for a multistart optimization [6]. This method avoids getting stuck on a suboptimal solution by conducting a series of local searches with randomized initial conditions. For our application, the values of P, $\{a_0, b_0\}$, and $\{a_e, b_e\}$ are randomized. Other metaheuristics could be employed, but this lays outside the scope of this work.

After this first step, the actuator is added to our system. The actuation optimization has its own loss function λ^* and bounding boxes, which are used in a new multistart optimization — only able this time to vary P_{act} and $\{a^*, b^*\}$. Once the best solution has been obtained, we compute the mean length of each bar and the mean position of each chassis point. In conjunction with the bars layout, this ensemble characterizes the synthesized kinematic diagram.

2.3 Loss Functions

2.3.1 Geometry

In order to evaluate the performance of a solution, our algorithm exploits a loss function λ, which associates a score to any given parameterization, and is defined as follows:

$$\lambda(P) = w_1 \cdot DC(P) + w_2 \cdot CC(P) + w_3 \cdot SC(P) + w_4 \cdot WC(P) \tag{3}$$

The members DC, CC, SC and WC are criteria which each evaluate a different aspect of the mechanism. For each criterion, its weight w_i allows the user to determine its importance during optimization.

DC is the Deformation energy Criterion. It was introduced by Avilés in [1]. As described prior, this criterion allows for the minimization of the deformation of the mechanism from one configuration to another:

$$DC = \frac{1}{L^2} \sum_{\{a,b\} \in \mathbb{E}} \sigma^2(l_{ab}) \tag{4}$$

With σ the standard deviation, $l_{ab}(k) = \sqrt{(p_{a,1,k} - p_{b,1,k})^2 + (p_{a,2,k} - p_{b,2,k})^2}$ the length of ab in configuration k, and L the characteristic length of our problem (used to remove the influence of scale on the optimization, it corresponds to the diagonal of the bounding box).

We add to this first element the Chassis deformation energy Criterion (CC):

$$CC = \frac{1}{L^2} \sum_{m \in \{a_0, b_0\}} \Big(\sigma^2(x_m) + \sigma^2(y_m)\Big) \tag{5}$$

With $x_m(k) = p_{m,1,k}$ and $y_m(k) = p_{m,2,k}$ the coordinates of point m. While close to DC, CC does not minimize the variation of distances, but the displacement of a_0 and b_0 between configurations. It is an innovation when compared to the previous DEM algorithms, which simply lock their positions beforehand.

As described in 2.1, a singularity occurs when two bars linked by a revolute joint become aligned, and this translates into a change of the sinus of the associated angle. We thus define a Singularity Criterion (SC) as follows:

$$SC = \sum_{(a,b,c) \in \mathbb{F}} \sigma^2(\operatorname{sgn}(\sin(\widehat{abc}))) \tag{6}$$

This criterion is also, to our knowledge, an innovation for the DEM. It allows for more complex geometries, without running the risk of encountering singularities.

λ counts a last member: the Weight Criterion (WC). Its value depends on the cumulated length of all bars of the geometry, and it aims at reducing the weight of the mechanism.

2.3.2 Actuator

Once the geometry has been optimized, the algorithm must place the actuator. The parameters found during the previous phase are considered as constants. It is also accepted that the range of the actuator is imposed by the choice of the component. The main objective of this phase thus becomes to find the positioning of the actuator which respects its dimensions while minimizing the effort it provides. To achieve this result, the loss function λ^* was devised:

$$\begin{aligned}\lambda^*(P_{act}) = w_5 \cdot DC^*(P_{act}) + w_6 \cdot CC^*(P_{act}) + w_7 \cdot SC^*(P_{act}) \\ +w_8 \cdot WC^*(P_{act}) + w_9 \cdot LC(P_{act}) + w_{10} \cdot FC(P_{act})\end{aligned} \tag{7}$$

DC^*, CC^*, SC^* and WC^* are the criteria of λ, modified for the actuator. CC^* monitors v instead of $\{a_0, b_0\}$, and ensures that the base does not move between configurations; DC^* and WC^* monitor $\{\{a^*, w\}, \{b^*, w\}\}$ instead of $\mathbb{E}$, and respectively ensure that the attaches linking the end to the mechanism do not warp and stay lightweight; SC^* monitors (a^*, w, b^*) instead of $\mathbb{F}$, and ensures that the attaches do not flip over.

Regarding the respect of the range of the actuator, the Length Criterion (LC) is used. It is equal to zero while l_{vw} stays between L_{min} and L_{max} (the minimum and maximum lengths of the actuator), but increases rapidly otherwise.

The last member, FC, is the Force Criterion. It is what allows us to minimize the efforts of the actuator during movement, and is expressed as follows:

$$FC = \frac{\max(F_{act})}{\max(F_e)} \tag{8}$$

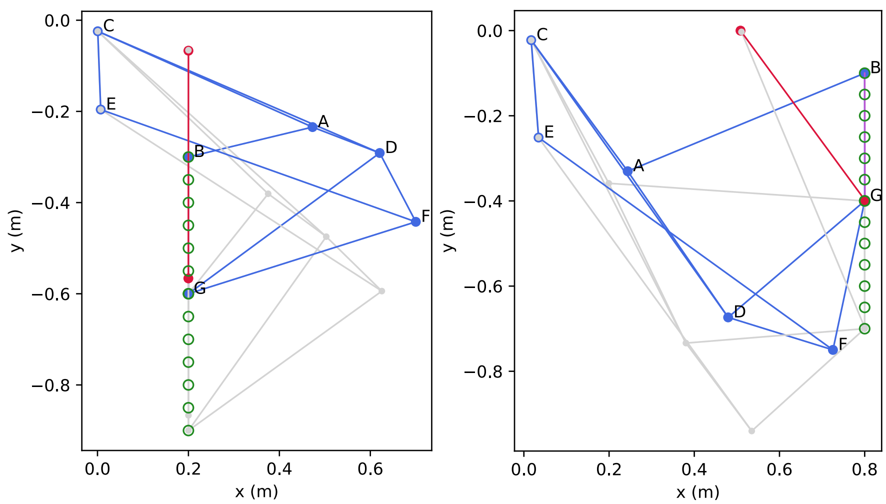
F_e is the force applied on the effector; it is determined by the user. F_{act} is the force applied by the actuator onto the mechanism, and can be computed via the works W_{act} and W_e. By supposing that the mechanism does not store energy (mass negligible when compared to the rest of the system), we get $\delta W_{act} + \delta W_e = \mathcal{P}\mathrm{d}t = 0$. Since $\delta W = \mathbf{F} \cdot \mathrm{d}\mathbf{u}$, if we make the hypotheses that $\mathbf{F_e}$ stays constant between two poses and that the effector realizes a translation, we can determine the effort provided by the actuator between the configurations i and $i+1$ (interval noted $i \rightarrow i+1$):

$$F_{act}(i \rightarrow i+1) = \left| \overrightarrow{F_e}(i \rightarrow i+1) \cdot \frac{\Delta x_{a_e} \cdot \overrightarrow{x} + \Delta y_{a_e} \cdot \overrightarrow{y}}{\Delta l_{vw}} \right| \tag{9}$$

3 Results

To assess the performance of our algorithm, a battery of tests was conducted for a panel of 4-bar and 6-bar topologies, with varying generation parameters (bounding boxes, trajectory, actuator length, etc.). The mechanisms presented below are its most indicative and pertinent results.

The risk of encountering singularities increases with the number of bars. We therefore chose to generate mechanisms from a Watt topology (a type of 6-bar mechanism), in hopes of testing the singularity criterion. Mechanisms were synthesized for two translational linear motions, placed differently inside of a 80×100 cm bounding box. The results can be seen in figure 2. The bars and joints of the mechanism are colored in blue, the actuator in red, the targets in green, and the attaches in purple (on the diagrams, they overlap with the bar BG). The first configuration of the mechanism is represented in color, while its last one is grayed out. It is important to note that in both cases A, C and D are part of a single body; the same goes for D, F and G.

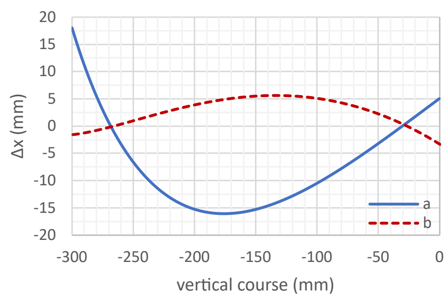


(a) Geometry I (b) Geometry II

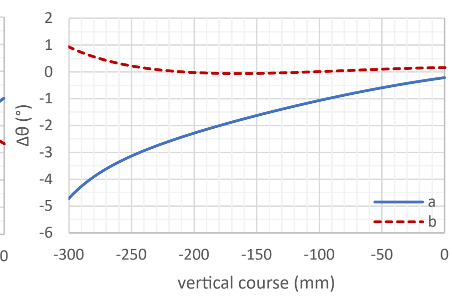
Fig. 2: 6-bar mechanisms optimized to follow different translations

First, the mechanisms do not encounter any singularity along their path. On tests where SC is removed, singularities appear and make the results unusable. This shows that SC fulfilled its purpose. Geometry I (figure 2a) presents a strong resemblance with the single-arm pantographs used on trains to maintain contact with overhead lines [2]. This is encouraging, since these systems are recognized as one of the best ways to approximate a linear motion with 6 bars.

A mechanical simulation software (MSC Adams) was used to compute the exact movements of geometries I and II. The resulting data is shown in figure 3. We can observe that geometry II stays very close to the target poses, deviating by less than 6mm (2% of the total course) horizontally and 1° in terms of orientation. Geometry I strays a bit further, with maximum errors of 18mm (6% of the total course) and 4.7°.



(a) Horizontal deviation of the effector



(b) Angular deviation of the effector

Fig. 3: Deviation from the target trajectory, for both 6-bar mechanisms

Regarding the placement of the actuator, we chose a model with $L_{min} =$ 50cm and $L_{max} =$ 80cm, applied a purely vertical effort on the effector, and set a 80×20 cm bounding box for the base (blocking it in the top section of the mechanism). The solution found for I is to align the actuator with the effector, and results in $FC = 1$. Since II lacks the space to do so while keeping its attaches small (weight criterion), it had to fall back on a configuration with $FC = 1.2$.

4 Conclusion & Perspectives

In this paper, we have presented a novel dimensional synthesis algorithm based on the deformation energy method. This algorithm provides innovations to the DEM on three main aspects: it allows for the optimization of the chassis, prevents the apparition of detrimental singularities, and integrates the optimal placement of an actuator into the synthesis process.

The first two points proved to be very successful during tests, resulting in the generated geometries being devoid of singularities and having a good quality of movement. The possibility to remove singularities with a lightweight criterion is deemed especially promising, as it opens up the synthesis method to topologies with a higher number of bars.

Meanwhile, the actuator optimization scheme managed to place actuators in a satisfactory manner, and can serve as a proof of concept. The authors are aware of the current limitations of this part — the restriction to a simple linear actuator without intermediary elements, the obligation to link it to the chassis, and the fact that the effort minimization does not take effector rotation into account — and are planning to make it more versatile in the future to expand its domain of application.

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