Application of Topological Requirements for the Definition of Orthosis Mechanisms

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Abstract This work explores the application of a novel mechanism synthesis methodology based on topological requirements for the systematic enumeration and filtering of hand rehabilitation exoskeletons. Traditional mechanism development often relies on designers' intuition, which limits the exploration of new structural possibilities and creates subjective processes. By applying a structured framework of topological requirementscategorized into degree (DR), subchain (SR), and relation requirements (RR)this study demonstrates how practical anatomical characteristics can be translated into computational filters. Crucially, these topological requirements act as mathematical guarantees that substructures inherent to human anatomy are preserved within the generated kinematic chains. Applied to a specific case study of a dorsal exoskeleton for digits 2 to 5, the established requirements ($M = 1$, $\nu = 5$, $\lambda = 3$) systematically reduced an initial set of 6,856 kinematic chains to 117 viable candidates. However, this represents just one possible configuration; by

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varying the orthosis-hand coupling strategy, the methodology can systematically generate entirely different atlases of structural possibilities, establishing a highly adaptable and computationally efficient design process.

1 Introduction

The rehabilitation of hand motor functions is a crucial aspect in the treatment of patients who have suffered strokes, spinal cord injuries, or other neurological conditions affecting upper limbs [1, 2, 3]. The implementation of robotic technologies in rehabilitation offers the possibility of extending treatment sessions for several hours, allowing for a greater number of exercise repetitions in an automated and assisted manner [4].

Despite advancements, the development of mechanisms traditionally relies heavily on designers' experience and intuition, creating dependencies that limit systematic innovation [5]. To overcome these limitations, structured methodologies for mechanism synthesis have evolved. Mechanism synthesis comprises three fundamental stages: number synthesis, type synthesis, and dimensional synthesis [6]. Number synthesis determines the structural configuration of candidate kinematic chains and defines the universe of possible solutions [7].

However, existing methodologies face significant challenges in efficiently filtering large sets of enumerated chains while maintaining practical relevance. Current approaches suffer from the absence of systematic frameworks for classifying observable practical requirements, increasing dependence on individual designer expertise and impeding the implementation of automated methods for kinematic chain filtering [8].

This work shifts the focus from the isolated development of a single exoskeleton device to the presentation of a robust methodological approach for enumerating viable orthosis topologies. By introducing topological requirements, we demonstrate how this methodology can systematically map the biomechanical complexity of the human hand into graph theory criteria. This allows for the generation of new, mathematically proven possibilities for hand exoskeletons based on varying anatomical coupling strategies [5] (Fig. 1).

2 State of the Art

2.1 Hand Exoskeleton Technologies

The field of hand rehabilitation robotics has seen a significant evolution in device types over the past decade. There are three main categories of devices: rigid exoskeletons, soft exoskeletons, and hybrid systems (Fig. 2). Rigid ex-

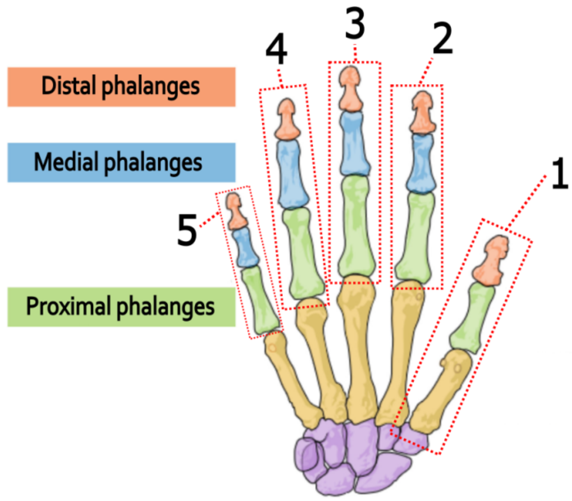

Fig. 1 Description of the components of the hand [5].

oskeletons have been the dominant design, accounting for approximately 50% of the studied devices [9]. These systems utilize rigid materials and linkage-based mechanisms, providing precise position control and robust force transmission [10].

Fig. 2 Device types [5].

2.2 Mechanism Synthesis Methodologies

Several authors have expanded on traditional synthesis methodologies to generate mechanical solutions. Approaches by Yan [11], Tsai [12], and Murai [13] have structured the enumeration process but lack rigorous delineation and categorization of practical mechanical characteristics [5]. Ding, Yang, and Kecskeméthy [8] proposed categorization into functional requirements and design constraints, yet it fails to define characteristics in an organized and replicable manner. Consequently, evaluating generated chains becomes susceptible to errors due to insufficient structured guidance toward critical local characteristics.

3 Methodology for Kinematic Chain Filtering

The methodology enhances traditional synthesis processes by introducing computational filtering tools that reduce subjectivity while maintaining practical relevance [5]. As represented in Fig. 3, it consists of nine stages, focusing heavily on Stage 4 (Number synthesis) and Stage 5 (Evaluation of viable kinematic chains).

3.1 Topological Requirements Classification

The key innovation of this methodology is the introduction of topological requirements as a distinct filtering category. Unlike traditional structural requirements (mobility, circuits and workspace), topological requirements capture local characteristics observable in successful mechanisms and translate practical engineering insights into mathematically tractable filtering criteria [5]. They are organized into three hierarchical categories:

1. **Degree Requirements (DR)**: Focus on individual link characteristics, filtering chains based on the necessary complexity of reference (DRR), end-effector link (s) (DREi) and intermediate links (DRIi).
2. **Subchain Requirements (SR)**: Address higher-order structures. A complementary innovation introduces **C-paths**, defining isolated subchains consisting exclusively of binary links arranged in series. Shared subchains maintain connections with multiple mechanism branches to distribute loads or define complex kinematic relationships.
   - SRIi: Delimits isolated serial subchains (C-paths).
   - SRSSi / SRSPi: Delimits shared serial and parallel subchains.

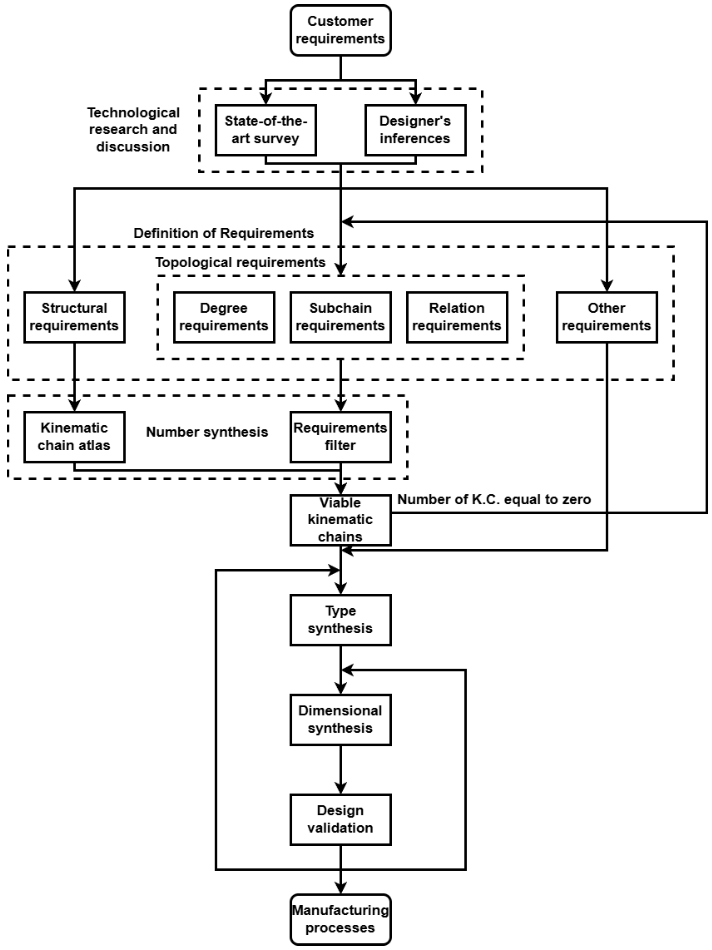

Fig. 3 Methodology used for the development [5].

3. **Relation Requirements (RR)**: Define mandatory connectivity patterns, ensuring proper integration of the previously specified elements. Specifically, this category formalizes reference-to-structure connections (RRRi), end-effector-to-structure connections (RREi), inter-subchain connections (RRSi), and general connectivity specifications (RRGi).

4 Application: Anatomical Mapping and Enumeration

To demonstrate the versatility of this methodology, it was applied to a specific coupling strategy for a hand orthosis. It must be emphasized that the chosen requirements below represent just *one possible approach* (a single-actuator dorsal mechanism coupled to three phalanges).

4.1 Translating Anatomical Needs into Requirements

For this specific case study, based on the review of state-of-the-art systems, structural requirements were set to Mobility $M = 1$, No. of circuits $\nu = 5$, and Workspace $\lambda = 3$. The topological requirements were carefully defined to mathematically guarantee the presence of structures inherent to this specific human anatomy coupling:

1. **Degree Requirements (DR)**:
   - DRR: The reference link must be ternary, providing adequate fixation to the hand's dorsum, with one conection to provide mobility.
2. **Subchain Requirements (SR)**:
   - SRSSa: A shared serial chain (3-4-3-2) must exist. This directly maps the physical structure of the finger phalanges (Fig. 4). The selection of a quaternary link (4) to abstract the proximal phalanx is a fundamental biomechanical requirement for dorsal exoskeletons. It must simultaneously interface with the metacarpal reference (3), the medial phalanx (3), the actuation subsystem, and accommodate passive alignment mechanisms (such as a Remote Center of Motion) to prevent hyperstatic shearing forces on biological joints without overconstraining the distal segments.
   - SRSPa: A shared parallel chain (3-2-2-3-3) must be present to guarantee the replication of an effective transmission subsystem, as this configuration accommodates a rotary actuator with a good lever arm and enables transmission from this closed sub-chain to multiple points across the equipment.
3. **Relation Requirements (RR)**:
   - RRSa: The subchains SRSSa and SRSPa must be connected by an intersection of the first link in the sequence, ensuring integration between the actuation system and the finger.

4.2 The Computational Filtering Process

An automated hierarchical filtering strategy was implemented using the SageMath graph library. The algorithm first applied degree filters through $O(1)$ partition-based evaluations, reducing the set to 4,212 graphs. Next, it utilized

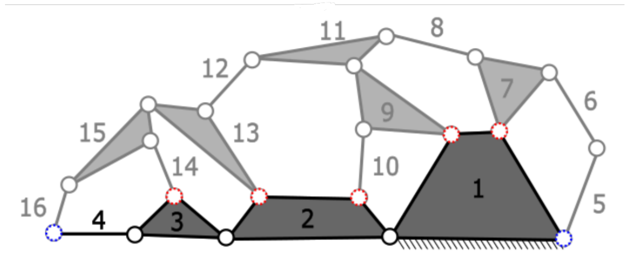

Fig. 4 SRSSa requirement reflecting a specific anatomical coupling sequence [5].

a polynomial-time directed neighbor-search heuristic to locate the $SRSS_a$ and $SRSP_a$ subchains, narrowing the candidates to 195 graphs. Finally, applying the relation requirement RRS_a via $O(1)$ cached index verification resulted in exactly 117 viable kinematic chains. This optimized approach completed the entire filtering process in 4.901 seconds.

5 Results - Exploring Adaptability and New Possibilities

The application of the methodology systematically reduced the initial set of 6,856 chains to 117 viable candidates, representing a reduction of approximately 98%. Within this final set, graphs from different partitions were retained, which accounts for the diverse topological characteristics observed, alongside structural variations present within each partition. Figure 5 presents four chains selected from this final set to illustrate this structural diversity across the different partitions. Furthermore, due to the imposed functional requirements regarding the number of circuits, mobility, and workspace, none of the 117 chains exhibit impractical topologies, such as long serial sequences of binary links. However, it is worth noting that for applications with higher-magnitude requirements, it would be advisable to implement an additional filter to eliminate chains containing binary sequences longer than three (e.g., a 4-path).

It is crucial to note that these 117 possibilities represent an atlas of solutions tailored strictly to the initial anatomical assumptions. The topological requirements applied here act as a *mathematical guarantee* that the inherent anatomical structure of the finger (in this case, three phalanges and a dorsal base) is preserved within the synthesized chains (as illustrated by the color-coded functional representation in Fig. 6).

However, multiple atlases of possibilities can be explored by altering the structural requirementsfor instance, by increasing or decreasing the number of independent circuits to investigate varying degrees of complexity. Furthermore, modifications to the mobility ($M = 2$) and workspace ($\lambda = 6$) allow

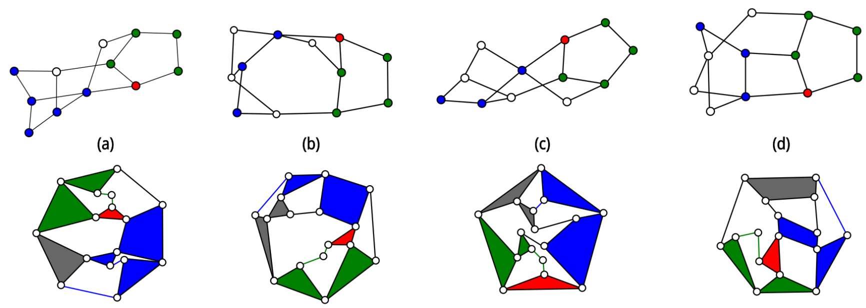

Fig. 5 Graphs and structural diagrams corresponding to the generated exoskeleton possibilities.

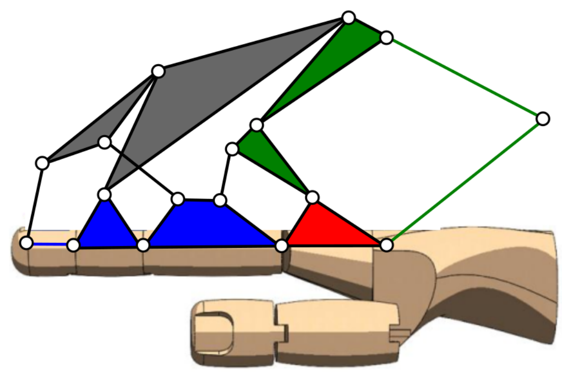

Fig. 6 Functional representation of a computationally generated mechanism candidate, demonstrating the embedded anatomical subchains.

for the analysis of finger adduction and abduction, extending beyond planar flexion and extension movements. Additionally, the selected topological requirements can be adjusted to design exoskeletons with either two or three coupling points to the hand. For these attachments, it is mandatory to account for the inherent anatomical structure of the fingers, as demonstrated by the SRSSa requirement, which must be adapted according to the number of fixation points. Ultimately, alternative actuation methods and configurations, such as incorporating sub-chains on the dorsum of the hand, remain open for future investigation.

6 Discussion and Conclusion

The shift in focus from intuitive mechanism design to a systematic enumeration driven by topological requirements proves highly advantageous for the field of rehabilitation robotics. The ability to translate complex anatomical traits and coupling strategies into computational filters effectively bridges the gap between biological necessity and mechanical synthesis.

By formalizing degree, subchain, and relation requirements, this methodology provides a mathematical safeguard: it ensures that any generated mechanism intrinsically possesses the required anatomical subchains to function as a human-machine interface. The case study demonstrated a 98% reduction in candidate chains, proving the computational efficiency of the approach. More importantly, it establishes that by simply varying the topological requirements to reflect different types of orthosis-hand interactions, researchers can automatically generate vast, entirely new atlases of structural solutions. This provides a fertile, mathematically rigorous ground for innovation, offering diverse structural configurations that might otherwise be overlooked by traditional design methods.

It must be explicitly stated that this methodology addresses the number synthesis (or topological synthesis) phase exclusively. Therefore, the mathematical guarantee provided is strictly structural: it ensures that a specific sequence of links with the exact required degree connectivity exists within the graph. This guarantee is a necessary, yet not sufficient, condition for a viable physical device. Determining whether these theoretical links can be physically dimensioned to avoid self-collision, circumvent kinematic singularities, and transmit forces effectively falls strictly under the subsequent dimensional synthesis phase. By intentionally halting the process at topological enumeration, this methodology dramatically reduces the computational search space (from 6,856 to 117), providing a highly purified atlas for future geometric optimization and physical prototyping.

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