Kinematics of ATHENA, a Robot for Pancreas Surgery

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Abstract ATHENA is a hybrid robotic assistant aimed at improving pancreatic surgery. It combines a 3-DoF actuated parallel module with a passive spherical mechanism that mechanically enforces the remote-center-of-motion (RCM) constraint and guides the instrument. This paper derives a complete kinematic model using Studys kinematic mapping and a dual-quaternion formulation.

1 Introduction

Pancreatic cancer is still one of the deadliest malignancies and surgical intervention remains the most curative treatment for patients with localized disease. Robot-assisted minimally invasive surgery can improve ergonomics, dexterity and visualization, reducing conversion and blood loss compared with laparoscopic approaches [7]. A key element in the development of such a surgical robot is the fixed remote

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center of motion (RCM) compliance, which limits the burden on the abdominal wall [8]. This can be achieved mechanically or through control strategies [9, 10]. Accurate kinematic models are essential for workspace and assembly-mode analysis, singularity avoidance and model-based planning and control. To support the surgeon in the interventions on the pancreas, this paper introduces ATHENA (Fig.1), a parallel robot developed for robotic assisted minimally invasive surgery (see also [3, 11]).

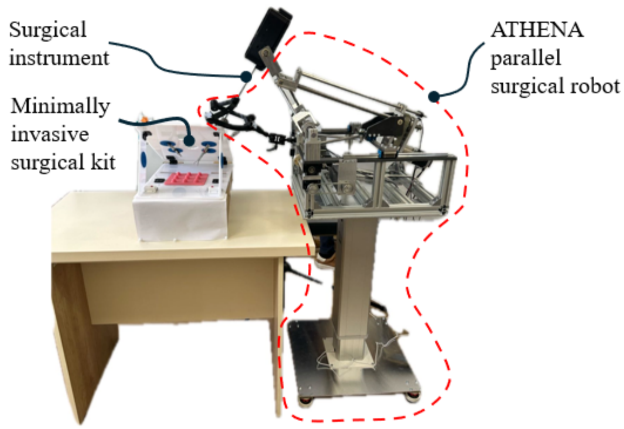

Fig. 1 Experimental model of the ATHENA parallel robot

ATHENA is a 3 DoF manipulator designed with modular mechanical elements, enabling to handle various instruments and perform multiple tasks in minimally invasive pancreatic surgery.

2 Mathematical Basics

For the mathematical description of the hybrid robot Athena, consisting of a parallel and a serial part and the derivation of its kinematic properties the Study parametrization of the Euclidean displacement group $SE(3)$, its interpretation as dual quaternions and some basic geometric considerations are used. Euclidean displacements are mappings

$$\gamma\colon \mathbb{R}^3 \to \mathbb{R}^3, \quad \mathbf{x} \mapsto \mathbf{A}\mathbf{x} + \mathbf{a} \tag{1}$$

where $\mathbf{A} \in$ SO(3) is a proper orthogonal three by three matrix and $\mathbf{a} \in \mathbb{R}^3$ is a vector. The entries of $\mathbf{A}$ fulfill the orthogonality condition $\mathbf{A}^T \cdot \mathbf{A} = \mathbf{I}_3$, where $\mathbf{I}_3$ is the three by three identity matrix. As usual, this mapping is written as a linear transformation with a four by four transformation matrix.[1]

[1] Note that in this paper the homogenizing coordinate is written as first coordinate.

$$\begin{bmatrix}1\\ \mathbf{x}\end{bmatrix} \mapsto \begin{bmatrix}1 & \mathbf{o}^T\\ \mathbf{a} & \mathbf{A}\end{bmatrix} \cdot \begin{bmatrix}1\\ \mathbf{x}\end{bmatrix}. \tag{2}$$

Study's kinematic mapping κ maps an element α of SE(3) to a point $X \in \mathbb{P}^7$. If the homogeneous coordinate vector of X is $[x_0 : x_1 : x_2 : x_3 : y_0 : y_1 : y_2 : y_3]^T$, the kinematic pre-image of X is the displacement α described by the transformation matrix

$$\mathbf{T} = \frac{1}{\Delta}\begin{bmatrix}\Delta & 0 & 0 & 0\\ p & x_0^2+x_1^2-x_2^2-x_3^2 & 2(x_1x_2-x_0x_3) & 2(x_1x_3+x_0x_2)\\ q & 2(x_1x_2+x_0x_3) & x_0^2-x_1^2+x_2^2-x_3^2 & 2(x_2x_3-x_0x_1)\\ r & 2(x_1x_3-x_0x_2) & 2(x_2x_3+x_0x_1) & x_0^2-x_1^2-x_2^2+x_3^2\end{bmatrix} \tag{3}$$

where

$$\begin{aligned} p &= 2(-x_0y_1+x_1y_0-x_2y_3+x_3y_2),\\ q &= 2(-x_0y_2+x_1y_3+x_2y_0-x_3y_1),\\ r &= 2(-x_0y_3-x_1y_2+x_2y_1+x_3y_0), \end{aligned} \tag{4}$$

and $\Delta = x_0^2+x_1^2+x_2^2+x_3^2$. The lower three by three sub-matrix of $\mathbf{T}$ is a proper orthogonal matrix if and only if $x_0y_0+x_1y_1+x_2y_2+x_3y_3 = 0$ and not all x_i are zero. When these conditions are fulfilled we call $[x_0 : \cdots : y_3]^T$ the *Study parameters* of the displacement α. Within this paper the homogeneous Study vector X will also be viewed as a dual quaternion and the matrix multiplication will be replaced by quaternion multiplication whenever it is more convenient.[2]

A serial linkage is described by a sequence of coordinate transformations

$$\mathbf{M}_1 \cdot \mathbf{G}_1 \cdot \mathbf{M}_2 \cdot \mathbf{G}_2 \cdot \cdots \cdot \mathbf{M}_n \cdot \mathbf{G}_n = \mathbf{E}, \tag{5}$$

where

$$\mathbf{M}_i = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \cos(u_i) & -\sin(u_i) & 0\\ 0 & \sin(u_i) & \cos(u_i) & 0\\ 0 & 0 & 0 & 1\end{bmatrix}, \qquad \mathbf{G}_i = \begin{bmatrix}1 & 0 & 0 & 0\\ a_i & 1 & 0 & 0\\ 0 & 0 & \cos(\alpha_i) & -\sin(\alpha_i)\\ d_i & 0 & \sin(\alpha_i) & \cos(\alpha_i)\end{bmatrix} \tag{6}$$

It has been shown in [4] that any serial kinematic chain can be brought into a pose where all rotation (or translation) axes are parallel to a plane. When this plane is chosen to be the yz-plane of the base coordinate system, then the successive coordinate transformations from one axis to the next can be described by the transformations $\mathbf{G}_i$ and the active rotations about the z-axes of the local coordinate systems are described by $\mathbf{M}_i(u_i)$. In the paper most times half tangent of the angles will be used written as $\tan\frac{\alpha_i}{2} = al_i, \tan\frac{u_i}{2} = v_i$. $\mathbf{E}$ is an arbitrary end-effector coordinate system.

[2] For more information on this approach see [2].

3 Forward Kinematics

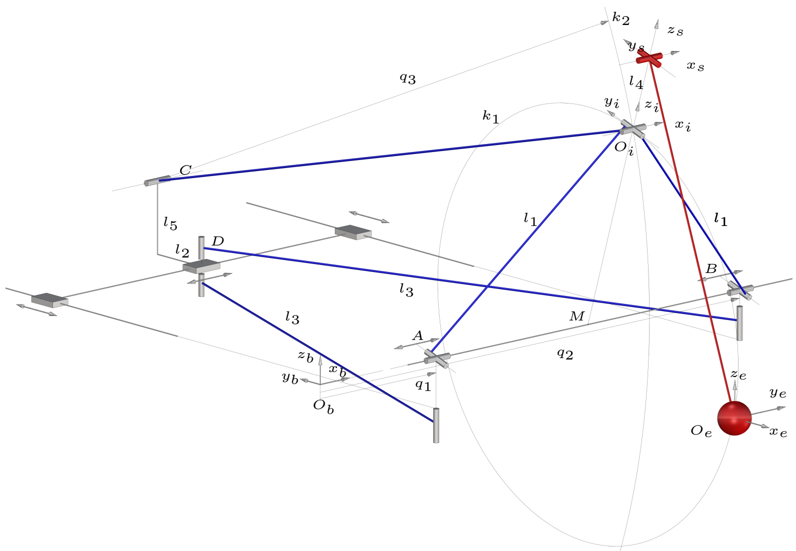

Fig. 2 Kinematic model of Athena

Fig.2 shows a kinematic model of ATHENA. The robot consists of two modules, namely a parallel robot in blue with three active DoFs [3] and a passive spherical mechanism. The passive spherical mechanism (Fig.3), presented in detail in [5], is to mechanically constrain the remote center of motion (RCM) of the instrument (the serial part in red), attached to the parallel robot. The passive spherical mechanism can be mechanically adjusted via two spherical joints (S_1, S_2). Its base coordinate system $O_e(x_e, y_e, z_e)$ is centered at the remote center of operation (RCM). The manipulation of the surgical instrument is performed using the parallel robot. The parallel robot module consists of five passive revolute joints, three passive prismatic joints and three passive universal joints depicted in grey. There are three active prismatic joints with active joint parameters $q_i, i = 1, \ldots 3$. Two active joints move the centers A, B of the two base U-joints along the x-axis of the base coordinate system. The third prismatic joint changes the radius of the circle k_2 centered at the point C. The geometrical design parameters are labeled $l_i, i = 1, \ldots 5$. For the kinematic analysis four coordinate systems are attached to the robot. The base coordinate system $O_b(x_b, y_b, z_b)$, the end-effector system of the parallel part $O_i(x_i, y_i, z_i)$ which is denoted by intermediate system. $O_s(x_s, y_s, z_s)$ is the base coordinate system for the serial part and $O_e(x_e, y_e, z_s)$ is the end-effector system of the serial part and

[3] A fourth active DoF, the rotation of the laparoscopic instrument, is not discussed in this paper because it does not contribute to the kinematics of the mechanism itself.

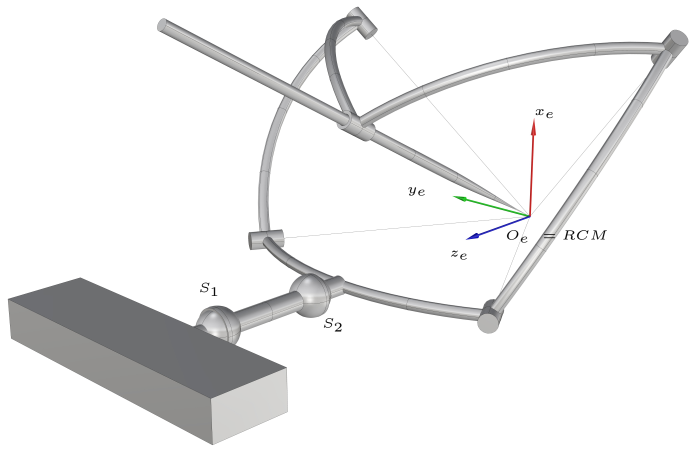

Fig. 3 Passive spherical five-bar

at the same time the coordinate system of the spherical five-bar that constrains the instrument.

3.1 Forward kinematics parallel part

In the forward kinematics of the parallel part, having the joint parameters q_i as inputs, the pose of the intermediate coordinate system $O_i(x_i, y_i, z_i)$ is to be determined. It is quite obvious that the four points C, D, M, O_i are always contained in a plane parallel to the $y_b z_b$-plane ($x = \frac{q_1+q_2}{2}$). Therefore the position of O_i can be found by the intersection of two circles k_1, k_2 contained in this plane. k_1 is centered at M, the mid-point between A and B and has radius $r_1 = \frac{\sqrt{4{l_1}^2-(q_1-q_2)^2}}{2}$. k_2 is centered at $C(my, l_5)$ where $my = \frac{\sqrt{4{l_3}^2-(q_1-q_2)^2}}{2} + l_2$ and has radius q_3.

$$k_1: \quad y^2 + z^2 = r_1^2, \qquad k_2: (y - my)^2 + (z - l_5)^2 = q_3^2 \tag{7}$$

The intersections of these two circles are possible locations of the origin of the intermediate coordinate system $O_i(x_i, y_i, z_i)$

$$x = \frac{q_1 + q_2}{2}, \qquad y = \frac{hmy^2 \pm \sqrt{w}}{2(my^2 + l_5^2)}, \qquad z = \frac{hl_5 \pm \sqrt{w}}{2(my^2 + l_5^2)} \tag{8}$$

where $h = (my^2 + l_5^2 + r_1^2 - q_3^2)$ and $w = -my^2(my^2 + l_5^2 - (r_1 + q_3)^2)(my^2 + l_5^2 - (r_1 - q_3)^2)$. It is easy to see that the orientation of the intermediate coordinate is just a rotation about the x_b axis. The y_i axis is tangent to the circle k_1 in O_i. With this information the coordinate transformation form the base to the intermediate system is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{q_2}{2} + \frac{q_1}{2} & 1 & 0 & 0 \\ \frac{hmy^2+\sqrt{w}}{2(my^2+l_5^2)} & 0 & \frac{hl_5+\sqrt{w}}{2(my^2+l_5^2)} & \frac{hmy^2+\sqrt{w}}{2(my^2+l_5^2)} \\ \frac{hl_5+\sqrt{w}}{2(my^2+l_5^2)} & 0 & -\frac{hmy^2+\sqrt{w}}{2(my^2+l_5^2)} & \frac{hl_5+\sqrt{w}}{2(my^2+l_5^2)} \end{bmatrix} \tag{9}$$

With this choice of the sign in the square root the intermediate coordinate system is oriented as in Fig.2. The second solution for the intersection of k_1 and k_2 is below the xy-plane and cannot be reached due to mechanical constraints most times. Both solutions coincide when $w = 0$. This concludes the forward kinematics of the parallel part.

3.2 Forward kinematics serial part

In the forward kinematics of the serial part the joint values of the passive joints of a serial UPS chain have to be determined. The base coordinate system for this chain is the system $O_s(x_s, y_s, z_s)$ which is obtained from the intermediate system $O_i(x_i, y_i, z_i)$ via a translation along z_i with translation distance l_4. Then the transformation from the base system to the start system of the serial chain is obtained:

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{q_2}{2} + \frac{q_1}{2} & 1 & 0 & 0 \\ \frac{hm_y^2+\sqrt{w}}{2(my^2+l_5^2)} & 0 & \frac{hl_5+\sqrt{w}}{2(my^2+l_5^2)} & \frac{(hm_y^2+\sqrt{w})l_4}{2(my^2+l_5^2)} \\ \frac{hl_5+\sqrt{w}}{2(my^2+l_5^2)} & 0 & -\frac{hm_y^2+\sqrt{w}}{2(my^2+l_5^2)} & \frac{(hl_5+\sqrt{w})l_4}{2(my^2+l_5^2)} \end{bmatrix} \tag{10}$$

Now the complete transformation from base to the end-effector system can be written

$$\mathbf{B} \cdot \mathbf{M_1} \cdot \mathbf{G_1} \cdot \mathbf{M_2} \cdot \mathbf{G_2} \cdot \mathbf{M_3} \cdot \mathbf{G_3} \cdot \mathbf{M_4} \cdot \mathbf{G_4} \cdot \mathbf{M_5} \cdot \mathbf{G_5} = \mathbf{E} \tag{11}$$

where the transformations are now written in dual quaternion form and the multiplication is quaternion multiplication. The Denavit-Hartenberg parameters of the serial chain are

$$\begin{aligned} DH :=& \{a_1 = 0, a_2 = 0, a_3 = 0, a_4 = 0, a_5 = 0, d_1 = 0, d_2 = 0, d_3 = p, \\ & d_4 = 0, d_5 = 0, al_1 = 1, al_2 = 1, al_3 = 1, al_4 = 1, al_5 = 0\} \end{aligned} \tag{12}$$

With this setup the surgical instrument is in the z-axis of the third coordinate system and the DH-parameter d_3 becomes a joint parameter denoted by p. The end-effector system $\mathbf{E}$ will be fixed in space by the surgeon using S_1 and S_2 and because of this all joints in the serial part are passive when the translational joints of the parallel part are given. Determining the values of the serial joints is therefore the inverse kinematics of the serial chain. Following the algorithm developed in [6] the serial chain is split into a left chain lc and a right chain rc:

$$lc = \begin{bmatrix} -2(b_0 v_1 v_2 + b_2 v_1 + b_3 v_2 + b_1) \\ -2(b_1 v_1 v_2 - b_2 v_2 + b_3 v_1 - b_0) \\ -2(b_2 v_1 v_2 - b_0 v_1 + b_1 v_2 - b_3) \\ -2(b_3 v_1 v_2 - b_0 v_2 - b_1 v_1 + b_2) \\ p(b_0 v_2 - b_3 v_1 v_2 + b_1 v_1) - 2(b_4 v_1 v_2 - b_2 p + b_6 v_1 + b_7 v_2 + b_5) \\ p(b_2 v_1 v_2 - b_0 v_1 + b_1 v_2) - 2(b_5 v_1 v_2 - b_3 p - b_6 v_2 + b_7 v_1 - b_4) \\ p(b_2 v_2 - b_1 v_1 v_2 - b_3 v_1) - 2(b_6 v_1 v_2 + b_0 p - b_4 v_1 + b_5 v_2 - b_7) \\ p(b_0 v_1 v_2 + b_2 v_1 + b_3 v_2) - 2(b_7 v_1 v_2 + b_1 p - b_4 v_2 - b_5 v_1 + b_6) \end{bmatrix} \tag{13}$$

$$rc = \begin{bmatrix} -e_3 v_3 v_4 v_5 - e_0 v_3 v_4 - e_0 v_4 v_5 + e_1 v_3 v_5 + e_2 v_3 - e_2 v_5 + e_3 v_4 + e_1 \\ e_2 v_3 v_4 v_5 - e_0 v_3 v_5 - e_1 v_3 v_4 - e_1 v_4 v_5 - e_2 v_4 + e_3 v_3 - e_3 v_5 - e_0 \\ -e_1 v_3 v_4 v_5 - e_2 v_3 v_4 - e_2 v_4 v_5 - e_3 v_3 v_5 - e_0 v_3 + e_0 v_5 + e_1 v_4 - e_3 \\ e_0 v_3 v_4 v_5 + e_2 v_3 v_5 - e_3 v_3 v_4 - e_3 v_4 v_5 - e_0 v_4 - e_1 v_3 + e_1 v_5 + e_2 \\ -e_7 v_3 v_4 v_5 - e_4 v_3 v_4 - e_4 v_4 v_5 + e_5 v_3 v_5 + e_6 v_3 - e_6 v_5 + e_7 v_4 + e_5 \\ e_6 v_3 v_4 v_5 - e_4 v_3 v_5 - e_5 v_3 v_4 - e_5 v_4 v_5 - e_6 v_4 + e_7 v_3 - e_7 v_5 - e_4 \\ -e_5 v_3 v_4 v_5 - e_6 v_3 v_4 - e_6 v_4 v_5 - e_7 v_3 v_5 - e_4 v_3 + e_4 v_5 + e_5 v_4 - e_7 \\ e_4 v_3 v_4 v_5 + e_6 v_3 v_5 - e_7 v_3 v_4 - e_7 v_4 v_5 - e_4 v_4 - e_5 v_3 + e_5 v_5 + e_6 \end{bmatrix} \tag{14}$$

b_i and e_i in lc and rc are the quaternions representing the intermediate (Eq.10) and the end-effector coordinate systems, v_i are the algebraic values of the joint parameters. lc and rc are equated to an unknown quaternion $X(x_0, x_1, x_2, x_3, y_0, y_1, y_2, y_3)$, which is representing the closure of both chains. This yields two sets of eight parametric quaternion equations in the unknowns x_i, y_i. Using the linear implicitization algorithm LIA developed in [1] the parametric equations are transformed into two sets of implicit equations via eliminating v_1 and v_2 from the left chain and eliminating v_3, v_4, v_5 from the right chain. The result are two sets sl and sr of implicit (polynomial) equations that contain the Study parameters x_i, y_i and one joint parameter p. All equations are linear in the Study parameters. Theoretically one could solve the system without assigning values to the intermediate and the end-effector coordinate system. But there is no gain because the equations get quite involved. So it makes sense to assign values to the quaternion of the intermediate and the end-effector system. Then from the set sl all y_i can be solved linearly. The results are substituted into the equations of the set sr. The four remaining equations in sr are still linear in x_i. One could be tempted to solve for all x_i but this is not possible because the Study parameters are homogeneous and this would result in $(0, 0, 0, 0)$ for all x_i. Therefore only three equations of the system are solved linearly for e.g. x_0, x_1, x_2. One Study parameter is still to be determined. In a general $6R$ one would use the Study equation to solve for this parameter. But in this very special chain containing a (passive) translational joint and a spherical part the Study equation is fulfilled automatically. Back substitution of the results for x_0, x_1, x_2 into the remaining equation of sr yields one quadratic equation for the joint parameter

p which can be solved closed form. The other joint parameters can be solved by substituting the values for p into the quaternions for the left and the right chain and equating $sc = \lambda rc$. This yields a system of eight equations for the remaining joint parameters. But this system is so simple that a Groebner base immediately yields the result, namely four solutions for each value of p. So all together there are eight solutions for the joint configurations of the serial chain. Together with the two solutions of the parallel part the complete manipulator has theoretically up to 16 solutions for the forward kinematics.

4 Inverse Kinematics

In the inverse kinematics of the parallel part an intermediate system $O_i(x_i, y_i, z_i)$ is given. Note that this coordinate system is not arbitrary. It must have the x-axis parallel to the x_b axis of the base system and the z-axis must intersect the x-axis of the base, because the circle k_1 is contained in the plane $y_i z_i$ and the y_i-axis is tangent to this circle. The x coordinate of the center M is therefore the x_i coordinate of the origin of the intermediate coordinate system $\frac{q_1+q_2}{2}$. The distance between M and B can be computed via the right angled triangle MBO_i and from this q_1 and q_2 are obtained. Via the triangle MBD the center C of the circle k_2 is computed and the distance between C and O_i yields the joint parameter q_3.

In the inverse kinematics of the serial part the joint angles of the spherical end-effector are given and the location of the $O_s(x_s, y_s, z_s)$ system is sought. This problem can be solved as a special backward problem of the UPS serial chain.

$$\mathbf{B} = \mathbf{E}\cdot\mathbf{G_5}^{-1}\cdot\mathbf{M_5}^{-1}\cdot\mathbf{G_5}^{-1}\cdot\mathbf{M_4}^{-1}\cdot\mathbf{G_3}^{-1}\cdot\mathbf{M_3}^{-1}\cdot\mathbf{G_2}^{-1}\cdot\mathbf{M_2}^{-1}\cdot\mathbf{G_1}^{-1}\cdot\mathbf{M_1}^{-1} \tag{15}$$

The right hand side of Eq.15 depends only at the angles v_1 and v_2 because v_3, v_4, v_5 are given. From the forward kinematics it is known that the matrix $\mathbf{B}$ has only a rotation about the x-axis. Therefore in the corresponding dual quaternion the third and the fourth entry must be zero. From this fact two equations in v_1 and v_2 are obtained. One of these equations is displayed:

$$\begin{aligned}
&e_3v_1v_2v_3v_4v_5 + e_0v_1v_2v_3v_4 + e_0v_1v_2v_4v_5 + e_0v_2v_3v_4v_5 - e_1v_1v_2v_3v_5 \\
&- e_1v_1v_3v_4v_5 - e_2v_1v_2v_3 + e_2v_1v_2v_5 - e_2v_1v_3v_4 - e_2v_1v_4v_5 + e_2v_2v_3v_5 \\
&+ e_2v_3v_4v_5 - e_3v_1v_2v_4 - e_3v_1v_3v_5 - e_3v_2v_3v_4 - e_3v_2v_4v_5 - e_0v_1v_3 \\
&+ e_0v_1v_5 - e_0v_2v_4 - e_0v_3v_5 - e_1v_1v_2 + e_1v_1v_4 - e_1v_2v_3 + e_1v_2v_5 \\
&- e_1v_3v_4 - e_1v_4v_5 + e_2v_2 - e_2v_4 - e_3v_1 + e_3v_3 - e_3v_5 - e_0 = 0
\end{aligned} \tag{16}$$

Both equations are bilinear in v_1 and v_2. They can be solved easily and yield a pair of solutions for v_1 and v_2. But these solutions determine only the orientation of the intermediate frame partially and its location is still open. A deeper inside into the meaning of the solutions shows that they determine two different frames.

These frames share the x-axis but they differ with a rotation about the x-axis with an angle of 180°. An easy geometric consideration shows that the intermediate coordinate system $O_i(x_i, y_i, z_i)$ also allows this rotation when the direction of the y_i-axis would have been chosen in the opposite direction. Combining both orientations in the correct way yields the same solution for the inverse kinematics of the parallel part!

There would be still six equations from Eq.15 to solve for the parameter p which determines the location of the frame. But all these equations are very complicated and contain square roots which make it difficult to solve. On the other hand, when the orientation of the end-effector is given, the orientation of the surgical instrument on which the origin of the $O_s(x_s, y_s, z_s)$ frame sits is determined. This means that the frame can only translate on this line and the correct location is such that the z_s-axis (no matter which of the two solutions is taken!) must intersect the x_b-axis. From this fact one obtains two linear equations which determine the location of the frame uniquely. Once the location of the $O_s(x_s, y_s, z_s)$ frame is found it can be translated back to the intermediate frame and the sought input parameters $q_1, q_2 q_3$ can be found as shown above. The solution is unique and this means that there is no inverse singularity in the parallel part and the two solutions for the serial part do not influence the solution of the parallel part. It is also important to note that the two solutions of the serial part can never coincide because they determine two disjoint sets in joint space. Two frames which differ by 180° can never coincide.

5 Conclusion

Using Study's kinematic mapping, dual quaternion representation and some basic geometric considerations the complete kinematics of the hybrid surgical robot Athena was developed. It turned out that due to the hybrid nature of the robot 16 solutions of the forward kinematics are possible. Furthermore it was shown, that in inverse kinematics, when the complete pose of the end-effector is given, the solution is unique and can be found via simple geometric reasoning. In the future the complete singularity set, forward and inverse, has to be determined to avoid different configurations of the robot.

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