Using Artificial Neural Networks to Solve Inverse Kinematics of Planar Flexible Manipulators

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Abstract This paper presents a neural network-based approach to solve the inverse kinematics of planar flexible manipulators. Multi-Layer Perceptron (MLP) neural networks are trained to predict the end-tip forces of each flexible rod as a function of their clamping angle and end-tip position. By decomposing the inverse kinematics problem into independent rod models, the proposed strategy enables an efficient search for static equilibrium configurations. The approach is demonstrated in a flexible planar $2-RFR$ parallel manipulator, showing that multiple inverse kinematic solutions can be identified with a reduced computational effort. Moreover, the method is not restricted to a specific mechanism, and it can be extended to more complex architectures.

1 Introduction

Collaborative robots (CoBots) have proven to be an effective tool for sharing tasks between humans and machines in many work environments [1]. At present, most CoBots are based on classical rigid robots that are endowed with flexibility through

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actuator control to increase their safety and adaptability. A more natural approach to CoBots is based on flexible robotics, showing new capabilities in safe physical interaction [2], although its technological maturity is still limited.

Within the field of flexible robotics, we can find the flexible or continuum parallel manipulators (PCMs). These are parallel manipulators in which some of the bars that form the kinematic chains are replaced by slender flexible bars, greatly reducing the overall morphology stiffness. PCMs are characterized by low inertia, high deformability, and intrinsic safety. Nevertheless, the development of flexible manipulators still poses significant scientific challenges, such as their nonlinear kinematic and dynamic modeling and the dependence of the model on the mechanical behavior of the material [3].

The modeling of large deformations can be approached using different methods, such as continuous models employing Cosserat rod theory and its derived Kirchhoff model [4], lumped-parameter modeling [5] and energy-based modelling strategies [6]. The Cosserat model involves the introduction of a system of ordinary differential equations (ODEs), whose solution entails a very high computational cost. Moreover, the proposed models have significant difficulties in identifying real parameters.

Recent advances in artificial intelligence enable the generation of hybrid numerical-AI models that make use of deep neural networks, such as Neural ODEs [7], allowing complex dynamic behaviors to be characterized and differential equations to be solved in a more efficient manner. In Cosserat rod based models [8], the integration of the system to get the shape of the flexible rodeither through numerical integration or using elliptic integralsis an unavoidable step that entails a significant computational cost. In a recent work [9], it was demonstrated that Neural ODE networks can be used efficiently to integrate the differential equations that govern the Elastica deformation.

The present work addresses the resolution of the inverse kinematic problem of planar flexible manipulators by means of neural networks of type MLP (Multi-Layer Perceptron). MLP networks have been already implemented in cable-driven mechanisms with promising results [10]. Here, we will use of the $2-RFR$ planar flexible manipulator as an illustrative example to show how to approach the inverse kinematics through MLP neural networks. In the proposed approach, the MLP of each flexible rod will work independently, and the combination of various MLPs will allow us to solve different types of mechanisms. Consequently, the method is not restricted to a specific mechanism.

2 Inverse kinematics: Planar Flexible Manipulator $2-RFR$

The planar parallel continuum mechanism shown in Fig. 1 has an end-effector at point P, where the end tips of two flexible rods are hinged and an external load is applied. The flexible rods are clamped to rotational actuators at A_i, $i = 1, 2$, and their large, nonlinear bending provides the mobility of the system. The inverse kinematics (IK) problem consists of finding the set of actuator input angles corresponding to given coordinates of point P, X_P and Y_P, and a specified applied load, such that the system is in static equilibrium. Therefore, there are two types of conditions to fulfill, i.e. a geometric assembly of both rods at P and a force balance of the system.

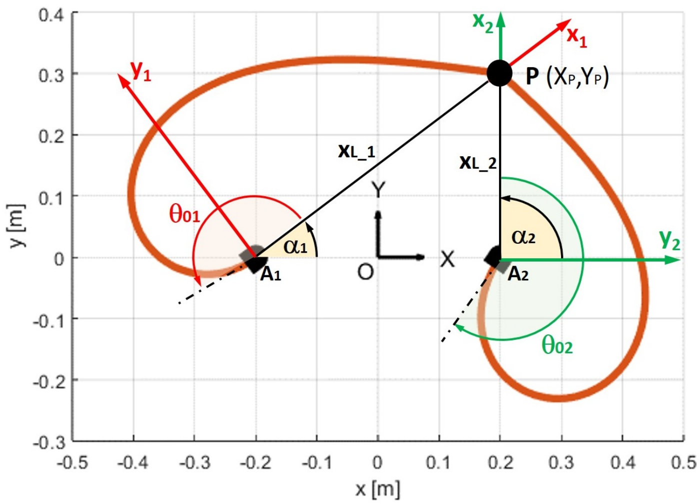

Fig. 1 Parameters of the IK problem

Our goal is to decompose the problem in such a way that the nonlinear behavior of each flexible rod can be solved independently, using the most comprehensive model possible and with minimal computational cost. This approach enables the extension of the method to mechanisms with higher degrees of freedom, a larger number of flexible rods, and different types of actuators or boundary conditions.

For the inverse kinematics (IK) problem, given the coordinates X_P and Y_P, the distances A_iP, denoted as x_{Li}, and the angles α_i can be determined, as shown in Fig. 1. First, we introduce a definition of local reference frames attached to the rods, which later allows us to generalize the solution for the deformed rod. The chosen convention is as follows: the x_i axis is defined from A_i to P, and the y_i axis is perpendicular to it and may be defined in either direction. In the example shown in Fig. 1, the reference frame of rod 1 is right-handed, whereas that of rod 2 is left-handed. All four possible combinations must be considered in the analysis in order to capture all multiple solutions of the IK problem.

Now, the deformed shape of each rod that satisfies the geometric conditions of the IK problem is defined by the clamping angle at the actuator, θ_{0i}, and by the local coordinates of the end tip at the hinge, x_{Li} and $y_{Li} = 0$. Therefore, inverse kinematics solutions can be obtained from the deformed shapes generated by rotating the clamped end of the rod while keeping the end-tip coordinate x_{Li} fixed, as shown in Fig. 2.

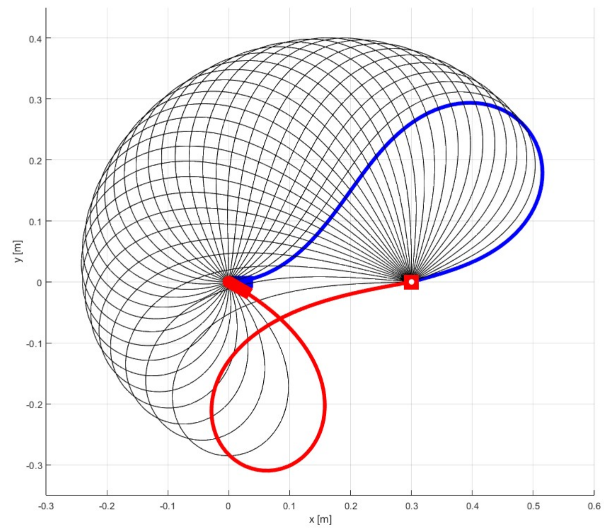

Fig. 2 Clamped-pinned Elastica solutions

The deformed shapes corresponding to IK solutions are those that satisfy the force balance condition. Consequently, the IK problem reduces to finding the clamping angles θ_{0i} of the rods, which are hinged at x_{Li}, such that they generate end-tip forces n_{xi} and n_{yi} that balance the external load applied at point P, taking into account the appropriate frame transformations:

$$\mathbf{F}_{ext} - \sum_{i=1}^{2} \mathbf{n}_i = \mathbf{0} \tag{1}$$

In order to identify all multiple solutions of the IK, we perform a search over all possible values of θ_{01} and θ_{02}, compute the corresponding end-tip forces $\mathbf{n}_1$ and $\mathbf{n}_2$, and evaluate the residual of Eq. 1. The combinations of θ_{01} and θ_{02} that make this residual zero correspond to valid solutions. Since the local reference frames attached to the lines A_iP may be defined as either right-handed or left-handed, four different combinations must be considered in the frame transformations applied in Eq. 1.

If a classical approach were applied, determining the rod shapes and the corresponding parameters, such as the end-tip forces $\mathbf{n}$, would require either solving a boundary value problem through direct integration of the rod equations using a shooting method, or formulating alternative optimization problems. These computationally intensive methods are also strongly dependent on the chosen deformation model, which may itself require refinement. The approach explored in this work instead relies on a neural network, which can be trained using theoretical models or experimental data and subsequently applied at a significantly reduced computational cost.

To begin with, we adopt the simplest neural approach: training Multi-Layer Perceptron (MLP) neural networks using data generated from the Kirchhoff rod model developed in [4]. To this end, we consider a rod clamped at $\theta_0 = 0$ with an end-tip located at x_L and compute the initial deformed shape, shown in blue in Fig. 2. The clamped end is then rotated clockwise to obtain the corresponding deformed shapes until loss of convergence is reached (red shape in Fig. 2); the same procedure can be applied in the counter-clockwise direction. All solutions within the range $[-2\pi, 2\pi]$ are combined with those obtained for different values of x_L in the interval $[0, L]$. This complete set of solutions is subsequently used to train the required neural networks. In a second stage, experimental data will be incorporated, allowing more complex material behaviors to be considered and non-physically feasible shapes to be discarded.

For the IK solving approach explained above, we only require an MLP to get end-tip forces $\mathbf{n}$ from input values θ_0 and x_L, as shown in Fig. 3. The MLP, which uses the two mentioned inputs and two outputs, is subjected to a hyperparameter optimization procedure using Hyperband. Through this process, the optimal hyperparameters resulted in a neural network architecture with five hidden layers, 128 neurons per hidden layer, and a *tanh* activation function. The dataset was split into training, validation, and test sets following a 70-20-10 ratio. It was normalized prior to training, and during the optimization process the Mean Square Error (MSE) loss was computed and minimized using the *AdamW* optimization algorithm until the lowest achievable MSE was reached. It is also worth noting that, for the time being, only clockwise rotation has been used to generate the training data. As a result, the range of solutions for θ_0 is reduced to $[0,\ 2\pi]$, and consequently some solutions may not be found.

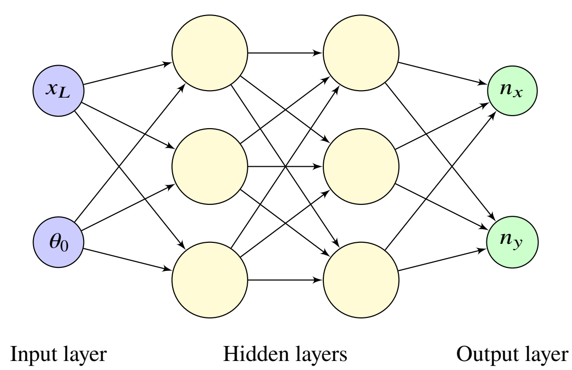

Fig. 3 MLP Neural Network for Rod internal force.

Once the MLP has been trained, it can be used to solve the inverse kinematics problem. From the given values of X_P and Y_P, the corresponding distances x_{L1} and x_{L2} are obtained, and the admissible ranges for θ_{01} and θ_{02} are identified based on the training data described above. A grid of discrete values is then defined in the θ_{01}-θ_{02} space for the search process. All combinations $(\theta_{0i},\ x_{Li})$ are evaluated with the MLP, and the residual of Eq. 1 is computed for the four possible frame transformation combinations. Figure 4 shows the residual plot for one of these cases, i.e. the frame orientation right-handed for rod 1 and right-handed for rod 2. In this example, only a single point $(\theta_{01},\ \theta_{02})$ yields a zero residual and therefore corresponds to an inverse kinematics solution for this frame arrangement. The same is applied to the other frame orientations to get all IK solutions. Please, be aware that the MLP was not trained with the whole space of possible θ_0 values, and therefore some solutions are missed.

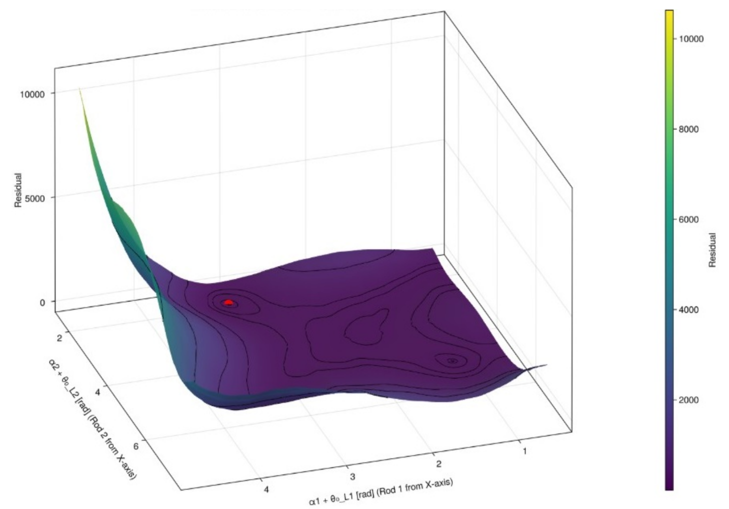

Fig. 4 Static Equilibrium Residual Function (Frames: Right-Handed, Right-Handed)

The values $(\theta_{01},\ \theta_{02})$ corresponding to inverse kinematics solutions are then combined with α_1 and α_2 to obtain the final solution sets $(\theta_1,\ \theta_2)$ expressed in the fixed reference frame. No further computation is strictly required to obtain the inverse kinematics solutions.

3 Rods' shapes for $2-RFR$ manipulator

Determining the shape of the mechanism for each inverse kinematics solution, or evaluating other related parameters, may be useful for design purposes. Once again, a neural network-based approach is adopted in order to solve this problem as efficiently as possible.

To reconstruct the rod shapes, we train an MLP network to predict the continuous deformed shape of a rod using the same training data obtained from the classical methods described above. A set of intermediate points along the rod is considered at arc-length positions s, for which the corresponding coordinates x_s and y_s are known. The MLP can be defined as in Fig. 5, with 3 inputs $(\theta_0,\ x_L,\ s)$ and 2 outputs $(x_s,\ y_s)$. The MLP, which uses the three previously mentioned inputs and two outputs, was subjected to a hyperparameter optimization procedure using Hyperband. Through this process, the optimal hyperparameters for the problem resulted in a neural network architecture with 3 hidden layers, 256 neurons per hidden layer, and a *gelu-tanh* activation function. The dataset was split into training, validation, and test sets following a 70-20-10 ratio. It was normalized prior to training, and during the optimization process the MSE loss was computed and minimized using the *AdamW* optimization algorithm until the lowest achievable MSE was reached.

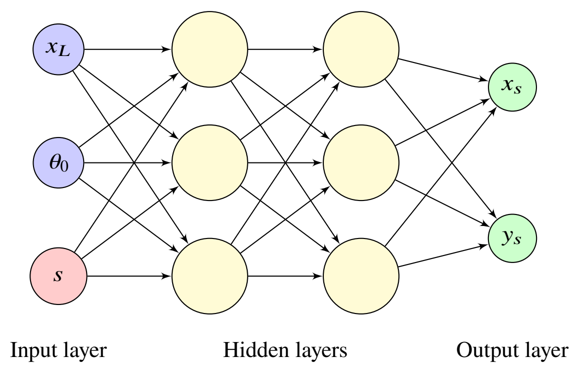

Fig. 5 MLP Neural Network for Rod's Shape.

Once the neural network is trained, it can be used to predict the deformed shape of each rod. The resulting shapes are then plotted in the global reference frame by applying the appropriate frame transformations to point A_i with angle α_i. For the inverse kinematics solution obtained above, the corresponding mechanism configuration is shown in Fig. 6. The continuous and dashed curves represent the real and approximated shape, respectively.

The same line of reasoning can be applied to estimate other rod parameters of interest. For example, an MLP similar to Fig. 3 can be trained to predict, instead of the internal force, the curvature at the clamped end of the rod, κ_0. This value can then be used to compute the rod shape through direct numerical integration of the rods differential equations. In this case, the problem is formulated as an initial value problem rather than a boundary value problem, resulting in a significantly faster solution. Moreover, this hybrid approach can yield more accurate results, since the MLP employed is simpler than the one used for direct shape reconstruction.

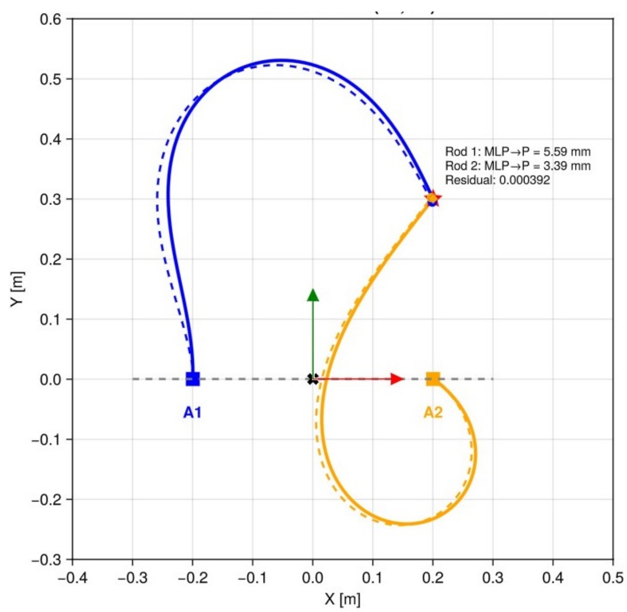

Fig. 6 IK solution found with Neural Network

4 Conclusions

This work demonstrates that MLP neural networks provide an efficient alternative for solving the inverse kinematics of planar flexible manipulators. By decoupling the nonlinear behavior of individual flexible rods, the proposed approach avoids the high computational cost and strong model dependence of classical methods. The results obtained for a $2-RFR$ planar flexible manipulator confirm that multiple inverse kinematics solutions can be identified through a static equilibrium search with significantly reduced computation time. Furthermore, since the MLPs could be trained using experimental data, the method offers a promising pathway to overcome modeling uncertainties and extend the approach to more complex mechanisms.

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